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Matrix-Based Modeling and Optimal Planning

By

Gozal Jafarli[1], Sudabe Alisoy[2] Ozcan Mammadov[3]

[1]Nakhchivan State University https://orcid.org/0009-0007-3743-5875
[2]Nakhchivan State University https://orcid.org/0009-0002-9559-6582
[3]Nakhchivan State University

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Abstract

The increasing complexity of modern systems necessitates the development of advanced mathematical frameworks capable of accurately modeling interdependencies among variables. Traditional optimization approaches, particularly linear programming models, often rely on simplifying assumptions that treat decision variables as independent. However, real-world systems inherently exhibit structural and functional dependencies among variables, which limits the adequacy of such classical formulations and calls for more sophisticated and flexible modeling approaches.

In this context, the present study introduces a matrix-based mathematical framework for optimal planning problems under constraints and extends classical optimization models through the incorporation of an interaction matrix. By explicitly representing interdependencies among decision variables, the proposed model provides a more comprehensive and realistic description of system behavior within a unified analytical structure.

The proposed approach is examined through the theoretical foundations of linear algebra, convex optimization, and spectral analysis. The analytical results demonstrate that the inclusion of the interaction matrix enhances both the structural expressiveness and the analytical depth of the model. Furthermore, it significantly improves the accuracy, stability, and practical efficiency of the optimization process.

Overall, the findings suggest that the proposed framework constitutes a meaningful advancement in optimization theory, offering a more robust and adaptable tool for modeling and solving complex constrained optimization problems (Boyd & Vandenberghe, 2004; Rockafellar, 1970).

INTRODUCTION

Optimization theory is one of the main directions of applied mathematics and plays an important role in the modeling of complex systems (Simon, 1996). Although classical linear programming models have a wide range of applications, one of their main limitations is the assumption of independence among decision variables (Dantzig, 1963; Bertsimas & Tsitsiklis, 1997).

In real systems, however, there exist interactions among variables, and these interactions directly affect system behavior. Since existing models do not adequately capture these relationships, the accuracy of optimization results decreases (Sterman, 2000).

Matrix theory provides an effective mathematical tool to overcome this problem. Expressing the relationships between variables and constraints in matrix form allows for a more precise structural modeling of systems (Bertalanffy, 1968; Strang, 2016).

The aim of this study is to construct a matrix-based optimization model that takes into account interactions among variables and to analyze its mathematical properties.

Mathematical Preliminaries

To establish the mathematical foundation of the proposed optimization model, we consider the general structure of linear systems defined in a finite-dimensional real vector space (Boyd & Vandenberghe, 2004; Strang, 2016). This formulation provides a rigorous analytical framework for representing relationships among variables and constraints, enabling a systematic and consistent description of the underlying optimization problem. Within this setting, linear transformations can be expressed in matrix form, which facilitates both the structural analysis of the system and the

*Corresponding Author: Gozal Jafarli

development of efficient computational methods for solving the optimization problem.

Assume that the system is modeled by the following matrix:
"A∈" "R" ^"m×n"

Here, matrix A acts as a linear operator representing the system constraints.

Decision variables are given by the vector:
"x∈" "R" ^"n"

This vector represents the controllable parameters of the system and serves as the main variable in the optimization process.

Constraints are defined as:

$$x \in R^n$$

which represents the physical, technological, or economic limits of the system.

The classical linear optimization problem is formulated as:

$$max f(x) = c^T x$$

subject to:

$A(x) \leq b,\ x \geq 0$

where $c \in R^n$ determines the coefficients of the objective function and defines the direction of optimization. This model belongs to the class of convex optimization problems, and its feasible region is characterized as a convex polyhedron (Boyd & Vandenberghe, 2004; Rockafellar, 1970).

Within this framework, the optimization problem aims to find an optimal point that maximizes the objective function within the feasible region.

However, one of the main limitations of this model is the assumption of independence among variables.

Theoretical Analysis

The inclusion of the interaction matrix in the optimization model introduces a fundamental transformation in its mathematical structure, effectively elevating it from the classical linear programming framework to a more generalized system-level formulation. This extension allows the model to capture intrinsic interdependencies among decision variables, which are typically neglected in traditional approaches.

Since the objective function remains linear, the convexity of the optimization problem is preserved. However, the geometric structure of the feasible region undergoes a significant transformation. Specifically, this structure is no longer determined solely by the constraint matrix A, but rather by the extended operator:
A+M

This matrix acts as the effective operator of the system, encapsulating both the original constraints and the interaction effects among variables. Its spectral properties play a central role in determining the fundamental analytical characteristics of the optimization problem, including stability, sensitivity, and the nature of feasible solutions (Strang, 2016; Luenberger & Ye, 2016).

In particular, the eigenvalues of the matrix A+M are critical in assessing system stability and solution behavior. If the matrix satisfies nonsingularity conditions and adheres to appropriate spectral constraints, the optimization problem can be considered well-posed, ensuring both the existence and stability of the solution. Conversely, unfavorable spectral properties may lead to instability, ill-conditioning, or multiplicity of solutions.

Furthermore, the incorporation of the interaction matrix transforms the optimization problem into a coupled system. In this context, each decision variable is no longer independent but is instead influenced by the entire system of variables. This interdependent structure provides a more realistic and comprehensive representation of complex systems, where such interactions are inherent and often essential for accurate modeling (Nocedal & Wright, 2006).

Discussion

The proposed matrix-based optimization model can be regarded as a significant and conceptually meaningful extension of classical linear programming approaches. While traditional models are fundamentally based on the assumption of independence among decision variables, the present framework explicitly incorporates interactions among them. This transition from independent to interdependent variable structures represents an important advancement in the modeling of complex systems, where such interactions are intrinsic and often critically influence system behavior.

One of the primary strengths of this approach lies in its capacity to formally and rigorously represent structural relationships within the system. By integrating the interaction matrix into the model formulation, interdependencies among system components are explicitly embedded within the mathematical structure rather than treated as external or negligible factors. This leads to a more precise characterization of system dynamics and allows the optimization process to capture underlying structural complexities that are typically overlooked in classical formulations. Consequently, the model provides a more coherent, analytically consistent, and theoretically grounded framework for optimization.

Another notable advantage of the model is its broad applicability across a wide range of complex and interconnected systems. In particular, in environments where decision variables exhibit strong interdependencies—such as networked systems, economic models, and multi-component engineering systems—the proposed approach yields more robust, stable, and context-sensitive results. Compared to classical linear programming models, which may oversimplify such interactions, this framework produces solutions that more accurately reflect real-world system behavior (Sterman, 2000; Bertalanffy, 1968).

Despite these advantages, the model is not without limitations. The most significant challenge arises from the increase in

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computational complexity associated with the inclusion of the interaction matrix. Specifically, the model leads to higher-dimensional and structurally more complex optimization problems, which may significantly increase computational cost and reduce the efficiency of conventional solution methods. This issue becomes particularly critical in large-scale systems, where the size and density of the interaction matrix can further complicate both analytical and numerical solution procedures.

Addressing these challenges requires the application of advanced numerical techniques and modern optimization algorithms capable of handling high-dimensional and structurally complex problems. Methods such as iterative optimization schemes, decomposition techniques, and scalable numerical algorithms are essential for ensuring the tractability and practical applicability of the proposed framework (Nocedal & Wright, 2006). Therefore, the development of efficient computational strategies remains a key direction for future research in this area.

Conclusion

In this study, a matrix-based extended mathematical framework for constrained optimization problems has been developed. By incorporating an interaction matrix, the primary limitation of classical linear programming models-namely, the assumption of variable independence-has been effectively addressed, enabling a more comprehensive representation of structural and functional relationships among system components.

The theoretical analysis confirms that the proposed model offers a more realistic, flexible, and analytically robust formulation of optimization problems. By explicitly capturing interdependencies among decision variables, the model enhances both the precision and the interpretability of optimization outcomes, thereby improving the overall reliability of the solution process.

Furthermore, this framework provides a unified mathematical perspective that integrates concepts from linear algebra, convex optimization, and spectral analysis, contributing to a deeper theoretical understanding of complex optimization systems. In this regard, the proposed approach not only strengthens the existing theoretical foundations but also extends the applicability of optimization methods to more complex and interdependent systems.

From a methodological standpoint, this study represents a significant advancement in optimization theory and aligns with the ongoing development of modern optimization frameworks (Boyd & Vandenberghe, 2004; Luenberger & Ye, 2016). It also establishes a solid foundation for future research directions, including nonlinear extensions, stochastic formulations, and the development of efficient computational algorithms for large-scale systems.

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*Corresponding Author: Gozal Jafarli