Global Scientific and Academic Research Journal of Multidisciplinary Studies
ISSN: 2583-4088 (Online)
Frequency: Monthly
Published By GSAR Publishers
Journal Homepage Link- https://gsarpublishers.com/journals-gsarjms-home/

CRITICAL REVIEW OF FUNDAMENTAL CONCEPTS IN PHYSICS Part 4 – "Classical Origins of Quantum Superposition"

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Article History

Received: 15/08/2025
Accepted: 20/08/2025
Published: 23/08/2025

Vol – 4 Issue –8

PP: - 60-66

Abstract
According to the concepts of modern physics, the phenomenon of "superposition" of probability amplitudes is characteristic only of quantum processes in the micro world and has no classical analogue. In this part of the text we show that the phenomenon of "superposition" is characteristic not only of objects of the micro world, and a similar phenomenon can be introduced into the probabilistic method of describing processes in the macro world. We indicate the fundamental principles of probability theory, common to both the micro world and the macro world, according to which the phenomenon of "quantum superposition" seems to us to be a special case of these principles. The study conducted in this part of the text is a direct demonstration of the idea expressed in the first part, according to which, due to the fact that the transfer of the simplest principles of probability theory to a wider space of probability amplitudes was carried out in parallel with the formation of quantum-mechanical concepts, many mathematical principles used in the implementation of these concepts were completely unjustifiably declared an exceptional feature of the quantum nature of the micro world. The results of the analysis presented in this part of the text will facilitate an easier understanding of the details of the phenomenon of "quantum superposition" discussed in the fourth part than is possible on the basis of the results of the analyses presented in textbooks on quantum mechanics.

Index Terms- *Quantum computer; Quantum or classical superposition; Collapse of the wave function; Reduction of the wave function; Classical or quantum.*

INTRODUCTION

In theoretical representations of quantum mechanics, it is widespread and popular to assert that "quantum superposition" of probability amplitudes is a specific feature of quantum mechanics, due to the peculiarities of the micro-world and having no classical analog. As the main argument, they point to the fact that this phenomenon concerns probability amplitudes, which were introduced to describe precisely quantum processes of the micro-world that have no classical analog. Below we will show that this assertion is incorrect.

Let us list the assertions discussed in this part of the text:

1. Is it possible or not - when describing random events for macroscopic bodies, to introduce state vectors corresponding to probability amplitudes?

Answer - not only possible, but necessary;

2. Should the physical and mathematical principles of probability spaces be different for objects of the micro and macroworld?

Answer - the principles of probability spaces should be universal and should not differ when used in different mechanics;

3. Should the mathematical principles of "quantum superposition" of state vectors of quantum objects differ from the mathematical principles of superposition of state vectors of macro objects, e.g., coins and dice?

Answer - the principle of superposition of state vectors is a universal characteristic of probability spaces and it should be realized identically in classical and quantum mechanics;

CHAPTER I: Brief History of the Issues

The introduction of the superposition phenomenon into quantum-mechanical discussions was connected with de Broglie's consideration, according to which - every corpuscular micro object corresponds to a wave, whose period length and amplitude are related to the quantitative values of the physical characteristics of this object (see (de Broglie, 1970)). De Broglie's consideration was connected with empirical facts, according to which - electromagnetic radiation, along with wave properties, also possesses corpuscular properties. Somewhat later, more important empirical facts became known - diffraction and interference of flows of different quantum particles at apertures, confirming the universality of de Broglie's opinion. In quantum mechanics textbooks, one can read that when a flow of micro objects passes through two micro-apertures located at some distance from each other, then on a screen - behind these apertures - the same interference images of quantum particle traces are obtained as we observe in the case of waves on the water surface and when light passes through similar apertures.

On one hand, wave representations of light, and on the other - the repeatedly verified ancient Greek principle that the properties of the whole are determined by the properties of its constituent parts and that these properties of the whole should be attributed to its parts (see (Baghaturia et al, 2025a)), naturally led to the opinion about the correctness of de Broglie's consideration - the carriers of wave properties are not only flows of micro objects, but also these micro objects individually. This idea also turned out to be in unison with the basic principle of probability theory, according to which - observed regularities in a set of statistical data for identical objects should be attributed to individual objects in the form of probabilistic characteristics.

Based on this, the following assertion was introduced into quantum-mechanical reasoning: under physical circumstances of one type, micro objects behave like corpuscles and under these circumstances are corpuscles, and under circumstances of another type they behave like waves and under these circumstances are waves.

This assertion became the basis of many erroneous representations, including - about the quantum nature of superposition of probability amplitudes. Therefore, it will be necessary to understand the essence of this phenomenon as well. This question was discussed by us in (Baghaturia et al, 2025b), where it was shown that there are neither empirical nor theoretical grounds for attributing wave nature to quantum objects of the micro-world, including photons. This means that there are no grounds for introducing the principle of corpuscular-wave dualism into theoretical representations of quantum mechanics. In turn, this means that the principle of superposition of probability amplitudes should be connected not with the wave nature of these amplitudes, but with their probabilistic nature.

Taking into account what has been said, let us proceed to discussing the first question from the list given above. Let us begin the discussion with a remark: flows of micro objects for which diffraction patterns are observed (which were erroneously attributed the term "interference", see (Baghaturia et al, 2025b) for details), represent a collection of non-interacting or weakly interacting objects. When forming a diffraction image, each object in the flow performs independent motion, similar to dice when they are thrown simultaneously. Empirically, precisely this fact is confirmed:

If micro objects of the flow are directed to the apertures individually, with a delay corresponding to large time intervals, then when detecting traces of these particles on the screen, the following picture is obtained: individual micro-particles leave localized traces, which corresponds to their corpuscular nature. But at the same time, the total image of traces has the same spatial-diffraction forms as are obtained when they are launched as a flow. That is, between the traces of such a joint picture there exists only statistical unity, and not dynamic unity, which arises in waves of some medium, for example - in water.

In this case, the application of the ancient Greek principle - to attribute the properties of the whole to its parts - would no longer be justified, since the mentioned flow is not a whole conditioned by internal connections, but only a statistically unified set of particles, to which wave essence was groundlessly attributed.

But the interpretation of empirical facts, on the basis of which representations about wave-corpuscular dualism were built, was considered indisputable, and together with facts of discreteness - observed in atomic processes, required appropriate theoretical explanation. There arose a feeling that since all these phenomena manifest precisely in micro-world processes, the explanation of the essence of these phenomena should be carried out within the framework of a new conception, characteristic only of the micro-world. Many opinions were expressed. For example, Schrödinger believed that electrons in an atom form a cloud-like spatially distributed substance in which these particles acquire wave nature. This opinion was not shared by many, since in all acts of observation the electron always manifested as a point-like localized particle. However, at the same time, everyone agreed that without introducing wave functions it would be impossible to explain the essence of the phenomenon of "wave-corpuscular dualism". All this, of course, required the introduction of adequate mathematical principles, and Max Born indicated - how this can be realized:

Phenomena corresponding to wave nature and discreteness should be attributed not to the micro objects themselves in the form of their physical characteristics, but to the totality of statistical data of results of repeating events with the participation of these objects. And since this totality of results corresponds to random outcomes of individual events, these phenomena should be attributed to micro objects in the form of probabilistic characteristics. For this, the probability space - defined by ordinary numbers, must be expanded, on one hand, to hyper-numbers of matrix algebra, which will correspond to the phenomenon of discreteness, and on the other hand - to wave functions described by complex variables, which will

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correspond to the phenomenon of wave nature. With the help of quadratic forms of elements of this expanded space, it will be possible to return to ordinary numbers, i.e., to probabilities. Wave functions, written in the form of columns of matrix algebra, could be given the interpretation of probability amplitudes, which would successfully correspond to both the ideology of matrix algebra and the wave nature of corresponding random outcomes (see (Born, 1926)).

The formation of the logical chain corresponding to new ideas was completed when M. Born gave an interpretation of the Schrödinger equation (see (Schrödinger, 1926)) - as a dynamic equation for probability amplitudes. As a result, the probability space was expanded not only to abstract hyper-numbers - corresponding to matrix algebra, but also to an even more abstract space of complex numbers. Since probability and probability amplitude are abstract mathematical constructions, attributing wave properties to them did not require the existence of any really existing ethereal medium. However, such expansion of probability space can introduce into the corresponding mechanics such degrees of freedom that will require great vigilance in the physical interpretation of corresponding mathematical relations (and when such details are not given sufficient attention, the ground is created for the emergence of myths - see, for example, (Baghaturia et al, 2025b) and (Japaridze et al, 2022)).

But at the same time, it is necessary to say the following: the introduction of probability amplitudes for describing physical states was one of the most important facts from both physical and mathematical points of view. The fact is that the introduction of state vectors as a mathematical principle of probability theory allows describing statistical reality more perfectly than would be possible without these vectors.

We will engage in demonstrating this question in the next subsection, which will become an effective "key" to clarifying the essence of the phenomenon of "quantum superposition."

CHAPTER II: Origins of the Superposition Principle in the Case of Macroscopic Objects

Let us return to the problem with dice and coins, discussed in the second part of the text, and try to introduce state vectors for them - corresponding to probability amplitudes. By analogy with spin, on one side of the coin we assign the number (1/2) and on the other - the number (-1/2). Similarly, on the faces of the die we assign the numbers {+5/2; +3/2; +1/2; -1/2; -3/2; -5/2}. Let us begin the discussion with a simple observation - the set of all possible numbers that appear in the results of throwing on the upper face of stationary objects represents a complete set of mutually exclusive possibilities. This assertion, corresponding to empirical reality, can be represented in the form of a mathematical principle by introducing state vectors. In the case of a die, this can be done with the help of six columns of matrix algebra:

$$V_1 = \begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix};\ V_2 = \begin{pmatrix}0\\1\\0\\0\\0\\0\end{pmatrix};\ V_3 = \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix};\ V_4 = \begin{pmatrix}0\\0\\0\\1\\0\\0\end{pmatrix};$$

$$V_5 = \begin{pmatrix}0\\0\\0\\0\\1\\0\end{pmatrix};\ V_6 = \begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}; \qquad (1)$$

The column V_i - conditionally, we associate with the state of the die when, after stopping, the number (7/2 - i) appears on the upper side; i - takes integer values from 1 to 6. Similarly, we can introduce two state columns for the coin:

$$v_1 = \begin{pmatrix}1\\0\end{pmatrix}; \qquad v_2 = \begin{pmatrix}0\\1\end{pmatrix}; \qquad (2)$$

Also conditionally, we assign v_1 to the state of the coin when the number (1/2) appears on the upper side, and v_2 - when (-1/2) appears. The abstract columns V_i and v_n correspond to spatial "polarization states" of objects that we observe on the upper side of the object after throwing in Earth's gravitational field and stopping on a horizontal surface. In matrix algebra, these columns represent linearly independent objects forming an orthonormalized basis of the corresponding vector space of probabilities:

$$< \bar{V}_i\, |V_j > = \delta_{ij}\ ; < \bar{v}_n |\, v_m > = \delta_{nm}; \qquad (3)$$

When $i \neq j$ and $m \neq n$, matrix products of these columns give zero numerical values, which we call mutual orthogonality of columns. The indicated conditions reflect empirical facts of mutual exclusion of corresponding physical "polarization states" - if one of them is realized, then simultaneously with it none of the others can be realized.

Here it should be recalled that probability space is built on the basis of our expectations, and these columns - in our expectations, should reflect the potential possibilities of objects - to turn out in some concrete state. Therefore, these columns should be assigned to corresponding objects - as mathematical characteristics reflecting these "potential possibilities." To turn the exposition into a mathematical principle, we introduce "generalized state vectors" of these objects:

$$\Psi = \sum_{i=1}^{6} \Psi_i = \sum_{i=1}^{6} C_i\, V_i\ ; \quad \psi = \sum_{n=1}^{2} \psi_n = \sum_{n=1}^{2} c_n\, v_n; \quad (4)$$

We select coefficients C_i and c_n so, that these "state vectors" satisfy the "completeness condition" of potential possibilities:

$$< \bar{\Psi}\ |\Psi> = \sum_{i=1}^{6} C_i^2 = 1; \qquad < \bar{\psi}\ |\psi> = \sum_{n=1}^{2} c_n^2 = 1; \quad (5)$$

These conditions can be easily fulfilled using empirical data obtained as a result of phenomenological analysis of empirical results of random events with the participation of these objects. For example, in "game mode" (see (Baghaturia et al, 2025a)):

$$C_i^2 = 1/6 \ \text{и}\ c_n^2 = 1/2; \qquad (6)$$

Squares of individual terms from (4):

$$< \overline{\Psi_i}\,|\,\Psi_i > = < \overline{C_i\, V_i}\,|C_i\, V_i > = C_i^2;$$

$$< \overline{\psi_n} \mid \psi_n > = < \overline{c_n \, v_n} \mid c_n \, v_n > = c_n^2; \qquad (7)$$

can be interpreted as probabilities of event outcomes, and the terms themselves - as amplitudes of these probabilities. Based on the presented reasoning, we can draw a conclusion:

The summation rule introduced in (4) is a complete analog of the mathematical phenomenon of superposition of state vectors in quantum mechanics. This summation rule - as a mathematical principle of probability theory, could be introduced into theory independently of quantum mechanics. In such an approach - the principle of superposition of quantum mechanics should be regarded as a particular case of a general mathematical principle.

When constructing principles of probability theory, we must not forget that the probabilistic characteristics that we attribute to objects as their potential possibilities actually correspond to our expectations and therefore are only abstract mathematical characteristics. Indeed, no matter how carefully we study the physical characteristics of the coins and dice themselves, we will find nothing similar to what probabilities c_n^2 and C_i^2 could correspond to. What we will discover - are two faces in the case of a coin and six faces in the case of a die. To someone it might seem that the numerical value c_n^2 = 1/2 is due precisely to the two faces of the coin, and the value C_i^2 = 1/6 is due to the six faces of the die. However, this would be a delusion, since the potential possibilities of these objects when throwing them include not only these numerical values of probabilities, but also any others satisfying the completeness conditions (5). And indeed - we can choose such "mechanical tricks" of throwing coins and dice, during which these coefficients will acquire other numerical values. As for the number of faces, they of course participate in forming probability spaces, but only indirectly - they set the numbers of mutually exclusive states: in the case of a coin - 2, and in the case of a die - 6, but these numbers have no direct relation to the numerical values of c_n^2 and C_i^2. But at the same time - it is precisely to these states that probability amplitudes are attributed.

Note that all potential outcomes of events in our expectations exist simultaneously. However, it should be noted that the use of the term "simultaneously" does not imply the existence of any chronological order in probabilistic reasoning. A statistical set of all possible event outcomes can be formed over a long period of time, but when describing the set of these outcomes, no chronological order is assumed for the elements of the set. The most important feature of the probabilistic method of description is a characteristic detail:

The probabilistic method of description deals only with final results of events and does not concern the course of events, and assumes neither dynamic description of the course of events nor any chronologization of event consequences.

Using the example of events with the participation of coins or dice, let us indicate the detail due to which the probabilistic method of description does not assume studying the question in a chronological context. The fact is that corresponding events occur in Earth's gravitational field. In the presence of this field, these events are formed, and the outputs of these events are determined as probabilistic. Without the gravitational field, of course, there would be neither such events nor would the corresponding statistical set of results be formed, on the basis of which the probability space indicated above could be built. When carrying out these events, we can consider that Earth's gravitational field is spatially homogeneous and invariant in time. And since external physical circumstances remain unchanged, the static physical characteristics of objects - participating in these events, and statistical-probabilistic regularities - corresponding to repeating events, do not depend on time and are completely stationary. Therefore, in the probabilistic method of describing event outcomes, there is no need to introduce chronological characteristics of these events, and for the same reason, corresponding probabilities are assigned to objects as time-independent characteristics.

Summarizing the above, let us indicate the following: all probabilistic events with the participation of objects of the type of coins and dice occur under macroscopically repeating external conditions, and only those physical circumstances can change which we call acts of "throwing" these objects. One type of such acts we call the "game mode."

Note that our expectations about potential event outcomes exist only until a concrete result is realized. At the moment of observing an already realized concrete result, all our expectations disappear as simultaneously as they arise and exist in our imagination before the moment of observation. This phenomenon is a characteristic of the probabilistic method of description and will be equally present everywhere where the probabilistic method of describing event outcomes is used.

This same phenomenon - in quantum mechanics, is called "Collapse of the wave function." In addition, in quantum mechanics reasoning, the phenomenon of "Reduction of the wave function" is introduced, which is also a general characteristic of probability theory. And indeed, after the completion of an event, the disappearance of our expectations in the "collapse" phenomenon corresponds to the disappearance of the probabilistic status of the superposition sum of the generalized state vector introduced in (4). This disappearance occurs when a concrete result is realized, which represents the "reduction" phenomenon from the generalized state vector to a concrete particular one. In quantum mechanics, this concrete detail of collapse is called "reduction of the wave function," and obviously it completely agrees with the classical representation of the collapse phenomenon. Consequently, the quantum collapse phenomenon corresponds only to a particular case of a more general phenomenon corresponding to the probabilistic method of description, and not to the quantum-mechanical nature of the microworld. We will return to details of the question of "quantum superposition" in the 5th part of the text.

Let us use the method of mathematical parametrization adopted in quantum mechanics and write the "generalized superposition state vector" of the coin in the following form:

$\Psi(\theta) = [a \cos\theta\, \Psi_1 + b \sin\theta\, \Psi_2] = [a \cos\theta \begin{pmatrix}1\\0\end{pmatrix} + b \sin\theta \begin{pmatrix}0\\1\end{pmatrix}]$;

$a = \pm 1$; $b = \pm 1$; (8)

the parameter θ is defined in the interval [0; π/2]. A concrete numerical value θ_i corresponds to a definite set of statistical data of event outcomes, which is generated by repeating "throws" of a definite i-th type. Through phenomenological analysis of this set, the numerical value θ_i is determined. In the state vector $\Psi(\theta_i)$, the statistical weight of physical state Ψ_1 equals $\cos^2\theta_i$, and the statistical weight of physical state Ψ_2 equals $sin^2\theta_i$. The "game mode" corresponds to the value $\theta = \pi/4$. Since the realization of $\Psi(\theta_j)$ and $\Psi(\theta_i)$ implies different and mutually exclusive types of "throwing" acts, then for these state vectors $\Psi(\theta_j)$ and $\Psi(\theta_i)$, the orthogonality relation should be satisfied:

$< \overline{\Psi(\theta_i)} | \Psi(\theta_j) > = \delta_{ij}$; (8-a)

It is clear that state vectors written in the form (8) do not satisfy this condition. Therefore, in the recording format of type (8), one should use the verbal principle of fulfilling condition (8-a). Let us formulate the principle:

Whatever the spectrum of basis vectors of probability amplitudes - discrete or continuous, superposition state vectors corresponding to different sets of mixing coefficients will always correspond to macroscopically well-defined and mutually exclusive physical circumstances, according to the label of which corresponding repeating events are realized. If in physical circumstances of a definite i-th type a definite superposition vector $\Psi(\theta_i)$ is realized, then in these same physical circumstances no other vector $\Psi(\theta_j)$ can be realized.

Let us consider one more important detail connected with the question: how to construct superposition sums in the case of several objects. As we mentioned in the second part of the text (see (Baghaturia et al, 2025a)) - state vectors of individual objects are introduced by means of a statistical ensemble. We can obtain this ensemble in two ways: either - as a result of repeating events in macroscopically repeating physical circumstances, performed by one object many times, or - as a result of collective actions performed simultaneously by many identical objects in the same physical state. The principle of identical status of statistical data of these two different realities assumes that in both cases - both events corresponding to each individual object and repeating events of one object are independent. Based on this, the method of statistical description establishes equality between the states of sets of statistical results obtained in these two different realities. Nevertheless, from the point of view of principles of statistical description, these two realities are not completely identical, which we can easily demonstrate using the example of two coins.

For this, let us write the state vectors of each coin in the form of superposition sums:

$\Psi^{(1)} = \sum_{n=1}^{2} \alpha_n v_n$; $\Psi^{(2)} = \sum_{n=1}^{2} \beta_n v_n$; (9)

For visual demonstration, let us consider in "game mode" two coins of different colors. In state vectors, we mark the color difference with corresponding indices and write them in the following form:

$\Psi^{(1)} = (1/\sqrt{2}) \sum_{n=1}^{2} \Psi^{(1)}_{[n]} = (1/\sqrt{2})[\begin{pmatrix}1\\0\end{pmatrix}^{(1)} + \begin{pmatrix}0\\1\end{pmatrix}^{(1)}]$;

$\Psi^{(2)} = (1/\sqrt{2}) \sum_{n=1}^{2} \Psi^{(2)}_{[n]} = (1/\sqrt{2}) [\begin{pmatrix}1\\0\end{pmatrix}^{(2)} + \begin{pmatrix}0\\1\end{pmatrix}^{(2)}]$; (10)

If we throw these two coins simultaneously, the superposition sum of state vectors of the system will have the form:

$\Psi^{(1\,;\,2)} = \Psi^{(1)(2)}_{[1][1]} + \Psi^{(1)(2)}_{[1][2]} + \Psi^{(1)(2)}_{[2][1]} + \Psi^{(1)(2)}_{[2][2]}$; (11)

$\Psi^{(i)(j)}_{[m][n]}$- corresponds to the physical state of the two-coin system, in which: the index (i) and (j) indicate the color of the coin, and the index [m] and [n] indicate a certain number indicated on the upper side of the coin of the corresponding color. In "game mode" we easily find that all possible physical states from (11) are generated by all possible physical states of individual coins, which are formed independently of each other. As a rule, mathematical realization of this empirical fact is realized according to the rule of "direct multiplication" of state vectors of individual coins, which is defined by rules of tensor product:

$\Psi^{(i)(j)}_{[n][m]} = \Psi^{(i)}_{[n]} \otimes \Psi^{(j)}_{[m]} = \Psi^{(j)}_{[m]} \otimes \Psi^{(i)}_{[n]}$; (12)

The tensor products defined in this way satisfy the permutability condition:

$\Psi^{(i)(j)}_{[n][m]} = \Psi^{(j)(i)}_{[m][n]}$; (13)

The specified condition of permutability does not mean invariance with respect to the rearrangement of indices within one row, since corresponding state vectors $\Psi^{(i)(j)}_{[n][m]}$ and $\Psi^{(i)(j)}_{[m][n]}$, or $\Psi^{(i)(j)}_{[n][m]}$ and $\Psi^{(j)(i)}_{[n][m]}$ correspond to different physical states of the given system. Thanks to the different color of coins, we can easily distinguish these realities from each other, therefore (11) should be written in the following form:

$\Psi^{(1\,;\,2)} = [\begin{pmatrix}1\\0\end{pmatrix}^{(1)} \otimes \begin{pmatrix}1\\0\end{pmatrix}^{(2)} + \begin{pmatrix}1\\0\end{pmatrix}^{(1)} \otimes \begin{pmatrix}0\\1\end{pmatrix}^{(2)} +$

$+ \begin{pmatrix}0\\1\end{pmatrix}^{(1)} \otimes \begin{pmatrix}1\\0\end{pmatrix}^{(2)} + \begin{pmatrix}0\\1\end{pmatrix}^{(1)} \otimes \begin{pmatrix}0\\1\end{pmatrix}^{(2)}]/2$; (14)

For each term of this sum, using permutations of the factors, the permutability relations specified in (12) are realized. State vectors from (14) correspond to mutually exclusive physical states that should satisfy orthogonality relations:

$4 < \overline{\Psi}^{(i)(j)}_{[n][m]} \mid \Psi^{(i)(j)}_{[k][l]} > = < \overline{\Psi}^{(i)}_{[n]} \otimes \overline{\Psi}^{(j)}_{[m]} \mid \Psi^{(i)}_{[k]} \otimes \Psi^{(j)}_{[l]} > =$

$= < \overline{\Psi}^{(i)}_{[n]} \mid \Psi^{(i)}_{[k]} > \otimes < \overline{\Psi}^{(j)}_{[m]} \mid \Psi^{(j)}_{[l]} > = \delta_{nk} \otimes \delta_{ml}$

$= \delta_{nk}\, \delta_{ml}$; (15)

In the given scalar products of the system state vectors, the multiplication operation occurs only between the elements of the probability space of a given object:

$< \overline{\Psi}^{(i)}_{[1]} \mid \Psi^{(i)}_{[1]} > = < \overline{\begin{pmatrix}1\\0\end{pmatrix}^{(i)}} \mid \begin{pmatrix}1\\0\end{pmatrix}^{(i)} > =$

$= \ < \ (1;0)^{(i)} | \ \binom{1}{0}^{(i)} > \ = \ 1;$ (16.1)

$< \overline{\Psi}^{(i)}_{[2]} \mid \Psi^{(i)}_{[1]} > = < \overline{\binom{0}{1}^{(i)}} | \binom{1}{0}^{(i)} > =$

$= < (0;1)^{(i)} | \binom{1}{0}^{(i)} > = 0;$ (16.2)

and multiplication between state vectors of different objects - does not occur. This principle corresponds to a quite concrete empirical fact: observing the corresponding physical state of the *i*-th column of coin $\Psi^{(i)}_{[1]}$, we discover only this concrete state of the given coin and cannot discover the same coin in state $\Psi^{(i)}_{[2]}$ and cannot discover physical states of the second object. Mathematical realization of the first two empirical facts is accomplished by relations (16.1) and (16.2). In terms of this mathematical realization, we cannot achieve mathematical realization of the last two facts - orthogonality of $\Psi^{(i)}_{[1]}$ relative to vectors $\Psi^{(j)}_{[1]}$ and $\Psi^{(j)}_{[2]}$. Therefore, we limit the possibility of such multiplication verbally.

For demonstration of the above assertion about incomplete identity of two different realities of statistical ensemble realization - from the point of view of possibilities of mathematical realization, let us consider the completeness condition for state vectors in the mathematical parametrization described:

$$< \overline{\Psi}^{(1)(2)} | \Psi^{(1)(2)} > =$$

$$= |\Psi^{(1)(2)}_{[1][1]}|^2 + |\Psi^{(1)(2)}_{[1][2]}|^2 + |\Psi^{(1)(2)}_{[2][1]}|^2 + |\Psi^{(1)(2)}_{[2][2]}|^2 =$$

$= 1/4 + 1/4 + 1/4 + 1/4 = 1;$ (17)

Corresponding results of these mathematical relations are confirmed empirically as well. Let us see what happens if these coins are of the same color and we cannot distinguish them from each other. We can no longer distinguish physical states corresponding to state vectors $\Psi^{(1)}_{[n]}$ and $\Psi^{(2)}_{[n]}$, and in (11) - all terms should be replaced by index-free representations:

$\Psi^{(1\,;\,2)} = \Psi_{[1][1]} + \Psi_{[1][2]} + \Psi_{[2][1]} + \Psi_{[2][2]} =$

$= (1/2)\ [\ \binom{1}{0} \otimes \binom{1}{0} + \binom{1}{0} \otimes \binom{0}{1} + \binom{0}{1} \otimes \binom{1}{0} +$

$+ \binom{0}{1} \otimes \binom{0}{1}];$ (18)

In physical states of the two-coin system, we can no longer distinguish either physical states - corresponding to state vectors $\Psi_{[1][2]}$ and $\Psi_{[2][1]}$, or corresponding mathematical representations of these vectors:

$\Psi_{[1][2]} = \binom{1}{0} \otimes \binom{0}{1} = \binom{0}{1} \otimes \binom{1}{0} = \Psi_{[2][1]};$ (19)

As a result, we can rewrite (18) in the following form:

$\Psi^{(1\,;\,2)} = \Psi_{[1]} + 2\Psi_{[1][2]} + \Psi_{[2][2]} =$

$= (1/2)\ [\ \binom{1}{0} \otimes \binom{1}{0} + 2\binom{1}{0} \otimes \binom{0}{1} + \binom{0}{1} \otimes \binom{0}{1}\ ];$ (20)

In the case of parametrization by such mathematical principles, the completeness condition of the probability space of the system will take the form:

$$< \overline{\Psi} \mid \Psi > \ =$$

$$= (1/4)\ |\binom{1}{0} \otimes \binom{1}{0} + 2\binom{1}{0} \otimes \binom{0}{1} + \binom{0}{1} \otimes \binom{0}{1}|^2 =$$

$= \ (1/4) \ [1 \ + \ 4 \ + \ 1] \ = \ 3/2;$ (21)

From the obtained expression, it is seen that the interpretation of the basis of probability space is destroyed. And therefore, state vectors introduced by such principles no longer represent probability amplitudes. One can try to save the mentioned basis by introducing new superposition coefficients in (20):

$\Psi^{(1\,;\,2)} = \ K_1\ \Psi_{[1][1]} + K_2\ \Psi_{[1][2]} + K_3\ \Psi_{[2][2]};$ (22)

and the completeness condition will take the form:

$K_1{}^2 + K_2{}^2 + K_3{}^2 = 1;$ (23)

However, in this case, we no longer have any preliminary theoretical considerations about what relations the mixing coefficients K_i can satisfy - using which we could predict their concrete numerical values. Therefore, we are left with the possibility - to indicate coefficients K_i only experimentally. Here it should be noted that the physical mechanism that is outside our control and which in "game mode" controls individual movements of coins and event results will not be "sensitive" to changing the color of coins. Consequently, for both systems - colored and unmarked identical coins - the same relations will be satisfied:

$K_1{}^2 = K_3{}^2 = K_2{}^2/\,2;$ (24)

Which corresponds to relation (17) obtained with the help of corresponding mathematical principles. Therefore, one can unambiguously say that mathematical principles with the help of which probability space state vectors corresponding to (21) and corresponding probability weights are obtained will contradict both empirical data and fundamental principles of probability theory.

As we see from the above discussion - it is impossible to orthogonalize probability spaces of different coins by introducing indices to denote individual coins. However, introducing such indices gives the possibility - to factorize probability spaces of different coins, which allows controlling the independence of these spaces from each other. In turn, this creates the possibility - to verbally prohibit scalar products of state vectors of different coins:

$< \overline{\Psi}^{(1)}_{[1]} \mid \Psi^{(2)}_{[1]} >$ (25)

By equating which to zero, we should realize the orthogonality condition. Since this cannot be achieved by rules of standard matrix algebra, we are forced to use the rule of verbal prohibition.

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Based on the presented reasoning, one can conclude that it is necessary to introduce the following principle into probability theory:

For introducing state vectors into probabilistic description of multi-object systems, it is necessary that in probability space these objects be marked, regardless of whether we can mark them in reality or not. This will allow indicating that real physical states of these objects are mutually exclusive. In the language of probabilities, this means that under no physical circumstances can another object be discovered in one concrete object, and it doesn't matter whether these objects are indistinguishable according to our subjective considerations or objectively - for reasons independent of us, as happens, for example, with particles of the micro-world.

Taking into account the obtained results, we proceed to discussing the phenomenon of "quantum superposition," which we will conduct in part V of the text.

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