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Symmetry Groups mediated by GeoGebra in the Training of Mathematics Teachers in rural schools

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Abstract
This study aims to investigate a proposal for training mathematics teachers in rural education, focusing on the integration of GeoGebra based on the Theory of Objectification and the Fedathi Sequence. The qualitative research, through case study and document analysis, explores a geometric proposal in the continuing education of teachers, aligning pedagogical practices with the guidelines of the National Common Curricular Base and the Reference Curricular Document of Ceará. The didactic proposal combines the use of GeoGebra with a teaching posture in the classroom activity, aiming to strengthen the pedagogical practice of teachers and make mathematics teaching more dynamic and contextualized. The research highlights the importance of continuing education and the use of digital technologies in rural education, promoting a flexible and dynamic learning environment.

Keywords: *Theory of Objectification, Rural Education, GeoGebra, Fedathi Sequence.*

Introduction

The proposal for teaching and learning Mathematics in the rural context, occurs through the insertion of the Theory of Objectification (TO), based on the Fedathi Sequence (SF) teaching methodology with the support of the GeoGebra software, particularly involving the Symmetry of Euclidean Geometry, in addition to visual perception, can enable a teaching experience with Mathematics teachers from rural schools in the municipality of Quixeramobim, including the National Common Curricular Base (BNCC) and the Ceará Reference Curricular Document (DCRC).

It is thus clear that educational institutions still maintain the traditional pedagogical proposal, in which teachers are the protagonists, with the function of holding and delivering information [1], making learning mechanized and focused on copying and memorizing. The pedagogical proposal of digital technologies fits into a situation that we do not experience much in educational institutions, and can be used in a conscious and reflective way, in contrast to the decontextualized and fragmented approach of the traditional educational system.

Based on this understanding, we seek to actively collaborate in the construction of knowledge. We investigate the training of rural education teachers, recognizing it as a fundamental path to teaching that values rural diversity. It is crucial that their knowledge is recognized and integrated into pedagogical practices, both inside and outside of school. We advocate for an inclusive education that respects the singularities and specificities of each individual. This education must promote a comprehensive education that does not ignore the culture, values, and knowledge of communities [2]. It is necessary to strengthen the identities of each people, encouraging dialogue between global knowledge and local realities. In this way, we ensure that education is a universal right, offered with quality and relevance.

Given this context, when reflecting on the connection between the Fedathi Sequence (SF) and the Theory of Objectification (TO) in the contextualized use of GeoGebra in the classroom, the following research question is formulated: how to develop the understanding of Symmetry Groups present in sociocultural artifacts, using geometric objects in the training of Mathematics teachers in the Final Years of Elementary School in rural schools, based on the TO and the SF teaching methodology, in line with the BNCC and the DCRC, and with the help of the GeoGebra software?

Thus, the general objective of this research is to analyze the application of Symmetry Groups in the training of Mathematics teachers in the Final Years of Elementary School in rural schools in the municipality of Quixeramobim, Ceará, using the Objectification Theory (TO) and the Fedathi Sequence (SF) methodology, in line with the National Common Curricular Base (BNCC) and the Ceará Reference

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Curricular Document (DCRC), with the support of the GeoGebra software.

The Theory of Objectification and the Fedathi Sequence

The Theory of Objectification (TO), developed by Luis Radford in the first half of the 1990s, proposes that mathematics education be conceived as a dynamic and multifaceted enterprise, encompassing political, social, historical and cultural dimensions. Its objective is the formation of reflective and ethical individuals, capable of adopting a critical stance towards mathematical discourses and practices, which are historically and culturally constituted and are in constant transformation [3].

In addition to the TO, the inclusion of the concept of artifact is relevant. According to this theoretical perspective, an artifact is an element of the material (and conceptual) world that has been modified throughout history by human actions. The notion of artifact as a result of human historical action allows us to overcome the dichotomy between the internal and the external, the ideal and the material. In this sense, we deepen the conception of artifact introduced into activity as a movement in which the subject's consciousness interacts with social objects, transforming each other in this encounter [4]. This conception of consciousness understands it not as a passive field of contemplation, but as an internal movement generated by human activity itself.

From this perspective, we present the Fedathi Sequence (SF), a teaching methodology developed by Professor Hermínio Borges Neto. The SF was conceived in 1971 in the Mathematics Department of the Federal University of Ceará (UFC), when the creator began teaching in the Bachelor's Degree in Mathematics course, where he remained until 1996.

For two and a half decades, several questions supported and shaped the teaching sequence structured by [5], initially inspired by the McLean Sequence and later consolidated as the Fedathi Sequence.

Based on the stages of the scientific work of mathematicians, the SF is structured in four successive and interconnected phases: Position Taking, Maturation, Solution and Proof. According to [6], the student actively participates in the historical processes that humanity has gone through to understand mathematical concepts, without the need to replicate the millennia of historical evolution.

In this context, it is observed that [5] conceived SF as a teaching method in the classroom, aiming to provide students with the experience of a mathematician in solving problem situations. Although the methodology is inspired by the work of the mathematician, it is possible to adapt it by incorporating other concepts in addition to mathematical knowledge. The main focus, however, remains on the methodological approach aimed at teaching and the teacher's posture, considering the students' performance in the learning process.

<table>
<tr><td colspan="2">Level 0: Preparation – Teacher's didactic organization, with environmental analysis, theoretical analysis (Plateau) and preparation of the didactic session.</td></tr>
<tr><td>Level 1: Experience – Development and execution of the teaching session in the classroom.</td><td>1st stage: Taking a Position – Defining the Didactic Agreement and presenting a challenging situation.</td></tr>
<tr><td>Level 2: Joint Work – Development and execution of the teaching session in the classroom. (doing it together)</td><td>2nd stage: Maturation – problem solving by students, with the mediation of the teacher through questions of the following types: clarifying, challenging, stimulating.</td></tr>
<tr><td>Level 3: Production - presentation of constructed knowledge that must be sufficiently complex to encourage discussion between student-student, student-groups, student-teacher.</td><td>3rd stage: Solution – socialization and comparison of the results found by the students. Use of counterexamples and counterquestions to support successes and possible errors.</td></tr>
<tr><td>Level 4: Analysis – teaching posture and student posture in the face of the culmination of the teaching and learning processes.</td><td rowspan="2">4th stage: Test – the teacher formalizes and/or generalizes the mathematical model constructed by the student, mediated by the artifact.</td></tr>
<tr><td>Level 5: Artifact – tool used in the knowledge construction process to solve problems.</td></tr>
<tr><td colspan="2">Level 0.0: Assessment - should be viewed by the teacher as a welcoming act. It is essential to consider its forms, processes, tools, and strategies that meet the public's demands.</td></tr>
</table>

Table 1: Fedathi sequence in teaching action

The relationship between the motivations of the teacher's actions and the content of the activity, reflected in the structure of consciousness as the connection between sense and meaning, makes it essential to consider the mediation of language. Therefore, in order to understand the construction of teaching practice, it is essential to analyze the senses and meanings that the teacher attributes to his/her actions and activities in professional training.

In teaching practice, the teacher interacts with the disciplinary content through artifacts and, simultaneously, transforms this content while being transformed by it. This engagement of the teacher is an integral part of the process of construction and development of teaching practice.

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Training in peasant schools

The training path for rural education teachers is outlined by the main laws and regulations in force for this type of teaching in Brazil. The following section establishes a connection between the continuing education of rural teachers and the theoretical foundations of [8] and [7].

It is understood that such training develops throughout the career, ranging from academic learning to teaching practice. This process occurs in multiple dimensions, through interactions with colleagues and the socialization of knowledge. Learning about the profession is profoundly influenced by the experiences lived during academic training. The personal marks of social relationships can have a positive or negative impact on training and professional performance [9].

Training encompasses both the initial phase, which qualifies for the exercise of the profession, and ongoing training, understood as an ongoing process. According to [9], the educational path extends from academic training to the practical context, considering its various nuances and formal and informal spaces, as previously mentioned. We start from this premise to discuss the training of Rural Education teachers, their educational paths, pedagogical practice and the knowledge produced both in academia and in everyday experiences in social relations.

The training of these teachers must articulate theoretical and methodological support that allows the professional, in their pedagogical practice, to strengthen peasant culture and act as a mediator of systematized knowledge, integrating global knowledge with local knowledge, general culture with popular culture, and academic knowledge with peasant experiences. The objective is to provide training that enables teachers to articulate, in their practices, science and common sense, disciplinary knowledge and popular knowledge, contributing to the development of contextualized education.

Methodological Path

From the perspective of this research, qualitative research (Alves-Mazzotti, 1999) is adopted, in line with the research objective and in connection with the instruments and sources selected for data collection, characterizing it as a case study.

The qualitative methodology, of an exploratory and descriptive nature, makes it possible to obtain contextualized information and data, in addition to helping to identify problems related to the research object. In this research, we seek to understand the dimension of continuing education for teachers in rural schools, from the perspective of the teachers.

Thus, in order to achieve the specific objectives, a theoretical and methodological survey is carried out on the concept of the activities to be applied and the learning conditions. At this point, a connection is established with the Mathematics contents of the Final Years of Elementary School present in textbooks and scientific research published in the area of Mathematics teaching. Additionally, a bibliographic review will be carried out to examine authors already cited in the chapter on theoretical references, deepening the topics of Teacher Training, Fedathi Sequence, Objectification Theory, Symmetry Groups, GeoGebra and Rural Education.

In the document analysis, official documents are read, both at the national level, such as the 2017 National Common Curricular Base (BNCC), and at the state level, including the 2019 Ceará Reference Curricular Document (DCRC), with the aim of mapping the resources recommended for teaching Geometry in basic education in Elementary School. According to [11], this type of study aims to provide a general and approximate view of a little-explored topic.

The analysis focuses on identifying the resources suggested by official documents for teaching Geometry and verifying the presence of aspects related to Rural Education in the guidelines on the use of these resources. Identifying these elements is essential to understanding which resources are recommended to teachers and the ways in which they are used, enabling a comparison with the resources available in the teachers' educational system.

The educational proposal aims to train mathematics teachers in rural contexts, integrating GeoGebra software as a pedagogical tool. Through workshops and practical activities, teachers will explore GeoGebra's functionalities for the construction of geometric concepts and problem-solving, adapting them to the reality of the countryside. The training of mathematics teachers in rural schools can work on the creation of digital teaching materials, the use of visual resources and the promotion of investigative activities, encouraging collaboration and the exchange of experiences among teachers. The proposal seeks to strengthen the pedagogical practice of teachers, making mathematics teaching dynamic, contextualized and meaningful for rural education students.

Symmetry Groups

The study of symmetry, inherent to the observation of nature and present in various artistic and architectural manifestations since ancient times, found in mathematics a precise language for its formalization. Although the intuitive notions of symmetry date back to time immemorial, the development of Group Theory, which provides the mathematical framework for describing symmetries, flourished mainly from the 19th century onwards. The work of mathematicians such as Évariste Galois and Niels Henrik Abel, at the beginning of that century, laid the foundations for Group Theory by investigating the solvability of polynomial equations.

However, it was with Felix Klein and his Erlangen Program (Erlanger Programm), presented in 1872, that Group Theory assumed a central role in geometry. Klein proposed a new way of classifying geometries, not by the nature of the geometric objects themselves, but rather by the groups of transformations that made them invariant. This revolutionary perspective defined geometry as the study of invariant properties under the action of a group of transformations, unifying diverse geometries under the same theoretical structure [12].

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The teaching of symmetry has its roots in the historical development of mathematics, and is incorporated into school curricula as an essential area for the development of logical and spatial thinking. The inclusion of symmetry in mathematics teaching reflects the need to build a solid foundation of spatial understanding, which is fundamental for other areas of mathematics and for solving everyday problems [13].

Symmetries of the Square

A square has eight distinct symmetries:

Identity: A transformation that does not change the position of the square.

Rotations: Three counterclockwise rotations of 90°, 180°, and 270° around the center of the square.

Reflections about the perpendicular bisectors: Two reflections about the lines that pass through the midpoints of the opposite sides of the square.

Reflections about the diagonals: Two reflections about the lines that connect the opposite vertices of the square.

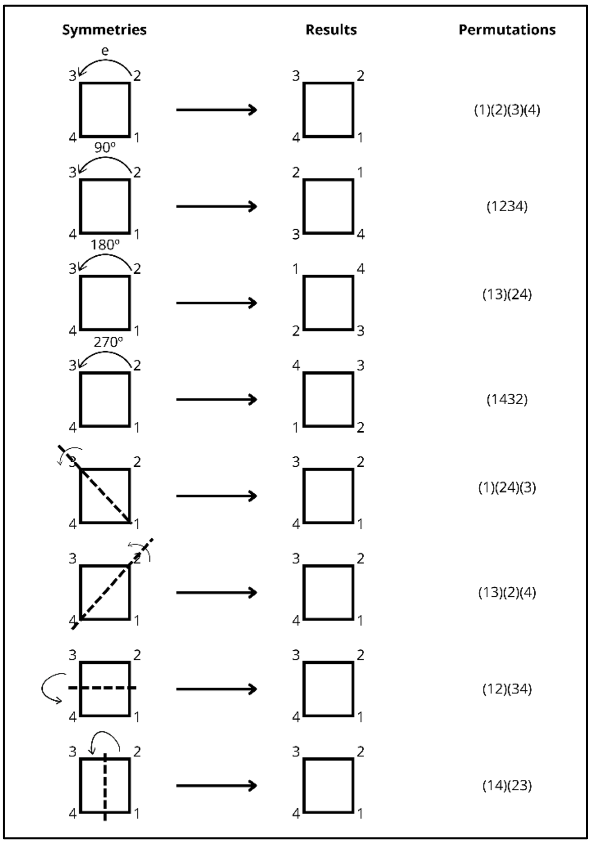

Figure 1: Symmetries, results and permutations of the square: Adapted from [14]

Given the vast range of theoretical elements presented, it is pertinent to demonstrate the path of using GeoGebra to explore Symmetry concepts. The presentation of a didactic trajectory is essential to support the objective of the research, which aims to integrate technological resources into the teaching of Symmetry.

GeoGebra in Teaching Symmetry

The GeoGebra software, developed by Markus Hohenwarter at the University of Salzburg, offers resources for various areas of mathematics [15]. In this context, the technological tool is considered a facilitator of teaching, and not an indispensable condition for the subject of Mathematics.

GeoGebra is a dynamic software that integrates geometry, algebra and calculus in an interactive virtual environment, and is widely used to improve students' mathematical understanding. In Brazil, the use of GeoGebra has been the subject of studies that explore its potential in the teaching and learning of mathematics. Its intuitive interface allows the interactive construction of geometric figures, the manipulation of algebraic expressions, the creation of function graphs, the analysis of statistical data and the exploration of calculus concepts, all in a dynamic and visual way.

This characteristic makes it a powerful pedagogical tool for teaching and learning mathematics at various levels, from elementary school to higher education, enabling students to explore abstract concepts in a concrete and experimental way. This constructivist approach, facilitated by GeoGebra, places the student at the center of the learning process, encouraging exploration, discovery, and the formulation of hypotheses, in line with the new guidelines for teaching mathematics.

GeoGebra can be used to promote interactive and visual teaching. [16] emphasize that the use of GeoGebra contributes to the construction of solid mathematical concepts, by allowing students to visualize and manipulate mathematical objects in a dynamic way. The term "dynamic" is associated with the notions of movement and transformation, allowing students to observe the constructions made and, thus, more easily understand the geometric behavior of the elements involved in this teaching-learning process.

From this perspective, [17] considers that the exploration of visual representations in the training of mathematics teachers, with the help of computational resources, and the application of the teaching phases provided for in the Fedathi Sequence, enable the systematic introduction of mathematical concepts, preserving the exploration of mathematical records. This approach promotes a critical-reflexive reflection in the visualization of mathematical objects constructed in GeoGebra.

Leaf symmetry

This practical activity encourages accurate observation and the development of spatial reasoning. Additionally, the comparative analysis of the sheets allows the introduction of concepts such as congruence and similarity.

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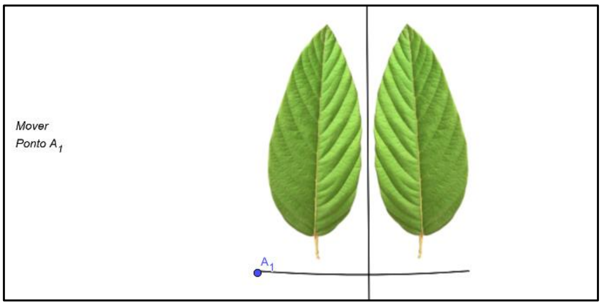

Figure 2: Symmetry in GeoGebra

The bilateral symmetry observed in leaves goes beyond mere aesthetics, playing a crucial functional role. The symmetrical distribution of veins, for example, optimizes the transport of water and nutrients throughout the leaf. Furthermore, the symmetrical shape can contribute to the efficient capture of sunlight. Understanding the intrinsic relationship between form and function in nature enriches learning and broadens students' view of the natural world. Observing nature, therefore, constitutes a valuable tool for understanding mathematical concepts.

Conclusion

The research explored the training of mathematics teachers in rural education, focusing on the integration of digital technologies, such as GeoGebra, and the foundation of the Theory of Objectification and the Fedathi Sequence. Ongoing training, combined with technological tools and innovative methodologies, can strengthen teachers' pedagogical practices, making mathematics teaching dynamic and contextualized for rural students.

The documentary analysis revealed the importance of aligning pedagogical practices with the guidelines of the National Common Curricular Base and the Ceará Reference Curricular Document, promoting an education that values rural culture and integrates local and global knowledge. The use of GeoGebra, as a visual and interactive tool, proved to be effective for the construction of mathematical concepts and for the development of students' spatial reasoning.

The didactic proposal presented, which combines GeoGebra with the Fedathi Sequence, offers a promising path for the training of mathematics teachers in rural education. We believe that the research contributes to the reflection on the importance of continuing education and the use of digital technologies in education, especially in challenging contexts such as the countryside, always seeking to build a flexible learning environment in the sociocultural context of the geometric objects of Symmetry.

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