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Lie group analysis of free convection flow over an inclined surface

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Abstract
Natural convection heat transfer past an inclined surface is investigated by Lie group analysis. Scaling symmetries are found and then the non-linear boundary layer equations with boundary conditions are transformed into a system of non-linear ordinary differential equations with appropriate boundary conditions. The resulting equations are solved numerically using the fourth-order Runge-Kutta scheme with Shooting method. Numerical results for the velocity and the temperature profiles are presented graphically for various values of parameters. It is observed that increasing Grashof number increases both the thermal and momentum boundary layer thicknesses and enhances the heat transfer rates. The velocity decreases with increase in the Prandtl number. Heat transfer rate decreases with Prandtl number.

Keywords*: Lie groups, Natural convection, Inclined surface, Boundary layer.*

I. INTRODUCTION

The study of natural convection flow for an incompressible viscous fluid past a heated surface has attracted the interest of many researchers in view of its important applications to many engineering problems such as cooling of nuclear reactors, the boundary layer control in aerodynamics, crystal growth, food processing, and cooling towers. In this paper, symmetry methods are applied to a natural convection boundary layer problem. The main advantage of such methods is that they can successfully be applied to non-linear differential equations. The symmetries of a differential equations are those continuous groups of transformations under which the differential equations remain invariant, that is, a symmetry group maps any solution to another solution. The symmetry solutions are quite popular because they result in the reduction of the number of independent variables of the problem.

Chen [1] performed an analysis to study the MHD natural convection flow over a permeable inclined surface with variable wall temperature and concentration. The results show that the velocity is decreased in the presence of a magnetic field. Increasing the angle of inclination decreases the effect of buoyancy force. Heat transfer rate is increased when the Prandtl number is increased. Dolapci and Pakdemirli [2] studied approximate symmetries of creeping flow equation of a second-grade fluid. They obtained approximate symmetries by three different methods. They concluded that in Method I in which basically dependent variables are not expanded in a perturbations series, rather the approximate generator developed in terms of the perturbation parameter fails to produce some approximate group-invariant solutions and Methods II and III are consistent with the perturbation theory and yield correct terms for the approximate solutions. They recommend Method III as the approximate symmetry method because Method II requires more algebra than Method III. Ibrahim et.al. [3] investigated similarity reductions for problems of radiative and magnetic field effects on free convection and mass-transfer flow past a semi-infinite flat plate. They obtained new similarity reductions and found an analytical solution for the uniform magnetic field by using Lie group method. They also presented the numerical results for the non-uniform magnetic field. Their results show that the velocity increases as the magnetic parameter and the Grashof number increase. When radiation parameter is decreased the temperature increases.

Kalpadides and Balassas [4] studied the free convective boundary layer problem of an electrically conducting fluid over an elastic surface by group theoretic method. Their results agree with the existing result for the group of scaling symmetry. They found the numerical solution also does so. The Navier-Stokes and boundary layer equations for incompressible flows were derived using a convenient coordinate system by Pakdemirli [5]. The results show that the boundary layer equations accept similarity solutions for the constant pressure gradient case. The importance of similarity transformations and their applications to partial differential equations was studied by Pakdemirli and Yurusoy [6]. They investigated the special group transformations for producing similarity solutions. They also discussed spiral group of transformations. Sanyal and Bhattacharyya [7] studied similarity solutions for natural convection of unsteady boundary layer MHD flow by group theoretic approach. They showed that similarity solutions are possible when the magnetic field is a constant or a function of x and t.

Using Lie group analysis, three-dimensional, unsteady, laminar boundary layer equations of non-Newtonian fluids are studied by Yurusoy and Pakdemirli [8, 9]. They assume that the shear stresses are arbitrary functions of the velocity gradients. Using Lie group analysis, they obtained two different reductions to ordinary differential equations. They also studied the effects of a moving surface with vertical suction or injection through the porous surface. They also studied exact solution of boundary layer equations of a special non-Newtonian fluid over a stretching sheet by the method of Lie group analysis. They found that the boundary layer thickness increases when the non-Newtonian behaviour increases. They also compared the results with Newtonian fluid. Yurusoy et al. [10] investigated the Lie group analysis of creeping flow of a second-grade fluid. They constructed an exponential type of exact solution using the translation symmetry and a series type of approximate solution using the scaling symmetry. They also discussed some boundary value problems. So far no attempt has been made to study the problem of the natural convection using Lie groups and hence we study the natural convection heat transfer flow past an inclined surface for various parameters using scaling symmetries.

2. MATHEMATICAL ANALYSIS

Consider the natural convection laminar boundary layer flow and heat transfer of an incompressible viscous fluid along an inclined semi-infinite plate with an acute angle α from the vertical. It is assumed that the thermophysical properties of the fluid are taken as constant. The governing equations of the mass, momentum, and energy for the steady flow can be written as [11- 15],

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)\cos\alpha \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with the boundary conditions

$$\begin{array}{lll} u=v=0, & (T=T_\omega) & at\ y=0, \\ u=0, & (T=T_\infty) & at\ y\to\infty, \end{array} \tag{4}$$

where u and v are velocity components; x and y are space coordinates; T is the temperature; υ is the kinematic viscosity of the fluid; g is the acceleration due to gravity; β is the coefficient of thermal expansion; k is thermal diffusivity and α is the angle of inclination.

The non-dimensional variables are

$$\bar{x}=\frac{xL_p}{u},\quad \bar{y}=\frac{yL_p}{u},\quad \bar{u}=\frac{u}{L_p},\quad \bar{v}=\frac{v}{L_p},\quad \theta=\frac{T-T_\infty}{T_\omega-T_\infty} \tag{5}$$

Substituting (5) into equations (1)-(4) and dropping the bars, we obtain,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+Gr\theta\cos\alpha \tag{7}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

with the boundary conditions

$$\begin{array}{lll} u=v=0, & \theta=1 & at\ y=0, \\ u=0, & \theta=0 & at\ y\to\infty, \end{array} \tag{9}$$

where $Gr=\frac{g\beta(T-T_\infty)u}{L_p^3}$ is the Grashof number and $Pr=\frac{u}{k}$ is the Prandtl number.

3. SYMMETRY GROUPS OF EQUATIONS

The symmetry groups of equations (6)-(9) are calculated using classical Lie group approach [16-20]. The one-parameter infinitesimal Lie group of transformations is defined as

$$\begin{aligned} x^* &= x + \epsilon\xi_1(x,y,u,v,\theta) \\ y^* &= y + \epsilon\xi_2(x,y,u,v,\theta) \\ u^* &= u + \epsilon\eta_1(x,y,u,v,\theta) \\ v^* &= v + \epsilon\eta_2(x,y,u,v,\theta) \\ \theta^* &= \theta + \epsilon\eta_3(x,y,u,v,\theta). \end{aligned} \tag{10}$$

By carrying out a straightforward and tedious algebra, we finally obtain the form of the infinitesimals as

$$\begin{aligned} \xi_1 &= 2c_1x - c_2x - c_3 \\ \xi_2 &= \tfrac{1}{2}c_1y - \tfrac{1}{2}c_2y - \alpha(x) \\ \eta_1 &= c_1u \\ \eta_2 &= -u\alpha'(x) - \tfrac{1}{2}c_1v + \tfrac{1}{2}c_2v \\ \eta_3 &= c_1\theta. \end{aligned} \tag{11}$$

Imposing the restrictions from boundaries and from the boundary conditions on the infinitesimals, we obtain the following form for equations (11)

$$\begin{aligned} \xi_1 &= 2c_1x - c_2x - c_3 \\ \xi_2 &= \tfrac{1}{2}c_1y - \tfrac{1}{2}c_2y \\ \eta_1 &= c_1u \\ \eta_2 &= -\tfrac{1}{2}c_1v + \tfrac{1}{2}c_2v \\ \eta_3 &= c_1\theta, \end{aligned} \tag{12}$$

where the parameters c_1 and c_2 represent the scaling transformations and parameter c_3 represents translation in the x coordinate. In the following sections, solutions corresponding to the above symmetries are derived

4. REDUCTION TO ORDINARY DIFFERENTIAL EQUATION

In this section, parameter $c1$ is taken to be arbitrary and all other parameters are zero in (12). The characteristic equations are

$$\frac{dx}{2x} = \frac{dy}{(1/2)y} = \frac{du}{u} = \frac{dv}{(-1/2)v} = \frac{d\theta}{0}. \tag{13}$$

From which the similarity variables, the velocities and temperature turn out to be of the form

$$\eta = x^{-1/4}y, \quad u = x^{1/2}F_1(\eta), \quad v = x^{-1/4}F_2(\eta), \quad \theta = F_3(\eta). \tag{14}$$

Substituting (14) into equations (6)-(8), we finally obtain the system of nonlinear ordinary differential equations

$$\begin{aligned} F_1'' &= \tfrac{1}{2}F_1^2 - \tfrac{1}{4}\eta F_1F_1' - GrF_3cos\alpha \\ F_2' &= \tfrac{1}{4}\eta F_1' - \tfrac{1}{2}F_1 \\ F_3'' &= Pr\left(F_2F_3' - \tfrac{1}{4}\eta F_1F_3'\right). \end{aligned} \tag{15}$$

The appropriate boundary conditions are expressed as

$$\begin{aligned} &F_1 = F_2 = 0, \qquad F_3 = 1 \qquad at\ y = 0, \\ &F_1 = 0, \qquad F_3 = 0 \qquad at\ y \to \infty, \end{aligned} \tag{16}$$

5. NUMERICAL METHODS FOR SOLUTIONS

Since the equations are highly nonlinear, a numerical treatment would be more appropriate. The system of transformed equations (15) together with the boundary conditions (16) is numerically solved by employing a fourth-order Runge-Kutta method and Shooting techniques with a systematic guessing of $F'(0)$ and $F'(0)$. The procedure is repeated until we get the results 13 up to the desired degree of accuracy, namely $10^{-5,}$ and solutions are presented graphically.

6. RESULTS AND DISCUSSIONS

Computations are carried out for various values of the Prandtl number Pr ranging from 0.005 to 5 and the Grashof number Gr from 0.1 to 2.5 with the angle of inclination α taking the values 0^0, $30^{0,}$ and 45^0. The numerical results are depicted in the form of velocity and temperature profiles. Most of the investigations are carried out for α = 45^0. Some results are taken for α = 0^0 (vertical plate case) and 30^0. Figure 1 shows the velocity profiles for *Pr* = 0.005 and various Grashof numbers. The velocity profiles along the surface increase with increase in Grashof number, that is, momentum boundary layer thickness is increased. Temperature profiles for *Pr* = 0.005 is depicted in Figure 2. From this Figure, it is seen that there is no noticeable change in temperatures when Grashof number is increased. Further increasing Prandtl number (= 0.1) increases the temperature, that is, thermal boundary layer thickness increases with increase in Grashof number, see Figure 3. In Figure 4, the behaviour of the velocity field is shown for *Pr* = 0.71 and different values of Grashof number. It is found that the velocity profile increases as Grashof number increases because of favorable buoyancy forces. Figures (5-6) show that the velocity profiles along the surface increase with increase in Grashof number while the temperature profiles also do so, both indicating that boundary layer thickness increases.

Figures 7 and 8 show the effect of Prandtl number on the velocity and temperature profiles along the surface for *Gr* = 2.5. Increase in the Prandtl number decreases velocity and temperature. So far the results are discussed for the case α = 45^0. Figures (9-12) show the velocity and temperature profiles for α = 0^0 (vertical surface case) and 30^0. The velocities for different values of parameters like Prandtl number and Grashof number in vertical surface case is higher than that corresponding to the inclined one. The effect of inclination on velocity and temperature profiles for various values of Prandtl number and Grashof number are drawn in Figures (13-16). Temperature inside the boundary layer for inclined surface is higher than that in the vertical one. Increasing the Grashof number decreases temperature inside the boundary layer. It is clear that increasing Grashof number or Prandtl number decreases the velocity of the fluid inside boundary layer, see Figures 13 and 15. For a given Grashof number, the velocity is decreased up to a certain level and then increased by increasing the angle of inclination. This is evident from the fact that the buoyancy effect decreases due to gravity components ($g cos\alpha$) as the plate is inclined.

7. CONCLUSIONS

By using Lie group analysis, we first find the symmetries of the partial differential equations and then reduce the equations to ordinary differential equations by using scaling symmetries. The resulting ordinary differential equations obtained by scaling symmetry together with boundary conditions are solved numerically. From the numerical results, it is observed that increasing the Grashof number increases both the thermal and momentum boundary layer thicknesses and enhances the heat transfer rates. It is observed that the velocity decreases with increasing Prandtl number. Also observed is that the thickness of the thermal boundary layer decreases with increasing Prandtl number.

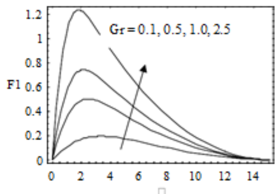

Fig. 1 The velocity profiles F1 along the surface for $\alpha = 45^{\circ}$, Pr = 0.005

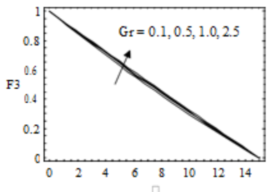

Fig. 2 The temperature profiles F3 along the surface for $\alpha = 45^{\circ}$, Pr = 0.005

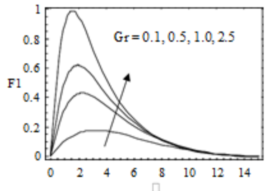

Fig. 3 The velocity profiles F1 along the surface for $\alpha = 45^{\circ}$, Pr = 0.1

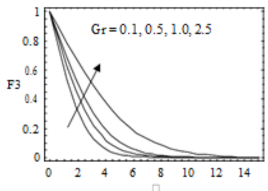

Fig. 4 The temperature profiles F3 along the surface for $\alpha = 45^{\circ}$, Pr = 0.71

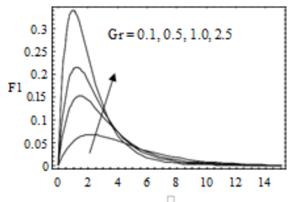

Fig. 5 The velocity profiles F1 along the surface for $\alpha = 45^{\circ}$, Pr = 5.0

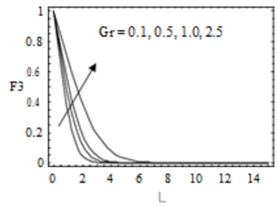

Fig. 6 The temperature profiles F3 along the surface for $\alpha = 45^{\circ}$, Pr = 5.0

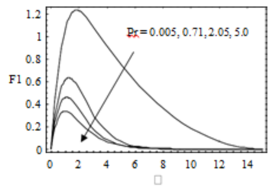

Fig. 7 The velocity profiles F1 along the surface for $\alpha = 45^{\circ}$, Gr = 2.5

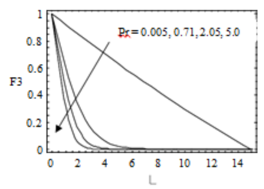

Fig. 8 The temperature profiles F3 along the surface for $\alpha = 45^{\circ}$, Gr = 2.5

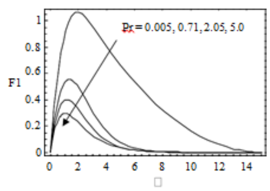

Fig. 9 The velocity profiles F1 along the surface for $\alpha = 0^{\circ}$, Gr = 1.0

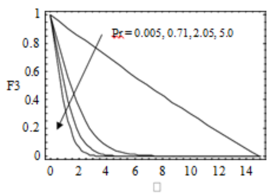

Fig. 10 The temperature profiles F3 along the surface for $\alpha = 0^{\circ}$, Gr = 1.0

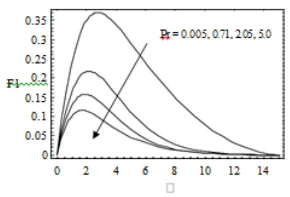

Fig. 11 The velocity profiles F1 along the surface for $\alpha = 30^{\circ}$, Gr = 1.0

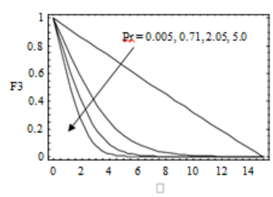

Fig. 12 The temperature profiles F3 along the surface for $\alpha = 30^{\circ}$, Gr = 1.0

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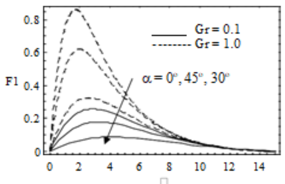

Fig. 13 The velocity profiles F1 along the surface for Pr = 0.1

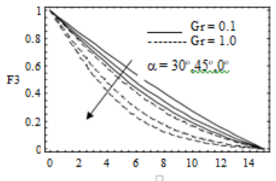

Fig. 14 The temperature profiles F3 along the surface for Pr= 0.1

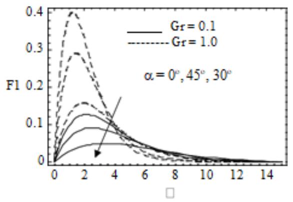

Fig. 15 The velocity profiles F1 along the surface for Pr = 2.05

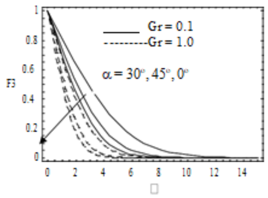

Fig. 16 The temperature profiles F3 along the surface for Pr = 2.05

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