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COMPLEMENT DEGREE POLYNOMIAL OF SOMEGRAPH OPERATIONS

By

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Abstract

$G = (V, E)$ be a simple undirected graph of order n and let $CD(G, i)$ be the set of vertices of degree i in complement graph $\overline{G}$ of G and let $Cd_i(G) = |CD(G, i)|$. Then complement degree polynomial of G is defined as

$$CD[G, x] = \sum_{i=\delta(\overline{G})}^{\Delta(\overline{G})} Cd_i(G)x^i.$$

In this paper, focus on complement degree polynomial of some graph operations and then derive some properties of this polynomial.

1. Introduction

Let G = (V, E) be a simple undirected graph of order n and let CD(G, i) be the set of vertices of degree i in complement graph G of G and let Cdi(G) = |CD(G, i)|. Then complement degree polynomial of G is defined as CD[G, x] = Δ(G) i=δ(G) $Cd_i(G)x^i$ [1].

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs, the union $G_1 \cup G_2$ is defined to be $G = (V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$, thesum $G_1 + G_2$ is defined as $G_1 \cup G_2$ together with all the lines joining points of V_1 to V_2. A self complementary graph is a graph which is isomorphic to its complement. Let v $V(G)$, then neighbourhood of v is defined as $N(v) =$ u : u is adjacent to v . The degree of a vertex v $V(G)$ is denoted by $deg(v)$.

In this paper, we derive complement degree polynomials of some graph operations and complement degree polynomials of some graph operations on path graph (middle graph, line graph, splitting graph, cosplitting graph, derivative of graphs, duplication of a vertex of a graph, total graph and central graph). The path graph is a tree with two nodes of vertex degree 1 and the other nodes of vertex degree 2. A regular graph is a graph where every vertex has the same degree. A regular graph with vertices of degree k is called k − regular graph. Let v ∈ V (G), then neighbourhood of v is defined as N (v) = {u : u is adjacent to v}.

2. Main Results

Theorem 1. *Let G be a graph with order n and $\mathbf{G} = G \cup G \ldots \cup G$ (mtimes). Then $CD[\mathbf{G}, x] = mx^{(m-1)n}CD[G, x]$.*

Proof. Observe that the order of G is mn. Let $v \in V(G)$ and degree of v is d, then $deg(v) = n - 1 - d$ in $\overline{G}$. Since v adjacent to each n vertices of $(m - 1)$ copies of G in **G** and v adjacent to the vertices $V - N(v)$ in G, $deg(v) = n - 1 - d + (m - 1)n$. Since v is an arbitrary vertex, $CD[\mathbf{G}, x] = mx^{(m-1)n}CD[G, x]$. This completes the proof. Q

Theorem 2. *Let G be a graph with order n and $H = G + G + \ldots + G$ (m times) . Then $CD[H, x] = mCD[G, x]$.*

Proof. Note that each vertex of H is adjacent to all vertices of $(m - 1)$copies of G. Therefore, each vertex $v \in V(mG)$ is adjacent to all vertices V $N(v)$. It follows that for each i, $Cd_i(H) = mCd_i(G)$. Hence $CD[H, x] = mCD[G, x]$. This completes the proof. Q

The coronagraph G H of two graphs G and H is defined as the graph obtained by taking one copy of G and $V(G)$ copies of H and joining thei^{th} vertex of G to every

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vertex in the i^{th} copy of H.

$CD[G \circ H, x] = x^{(n-1)m}CD[G, x] + nx^{(n-1)(m+1)}CD[H, x]$. *Proof.* Let G be a graph of order n and H be a graph of order m. Note that each vertex of G adjacent to m vertices of one copy of H. Let $v \in V(G)$ then v adjacent to $V(G) - N(v)$ vertices in G and $(n-1)m$ vertices of $(n-1)$ copy of H in $G \circ H$. Therefore, $deg(v) = deg(v)$ in $G + (n-1)m$ in $G \circ H$. Similarly, it is in the i copy of H, then u adjacent to $N(u)$ vertices in H and i^{th} vertex in G. Therefore, u adjacent to $V(H)$ $N(u)$ vertices in i^{th} copy of H, m vertices of n 1 copies of H and n 1 vertices of G. That is, $deg(u) = (deg(u)$ in $H)+m(n\ 1)+(n\ 1) = (deg(u)$ in $H)+(n\ 1)(m+1)$. Therefore, $CD[G\ H, x] = x^{(n-1)m}CD[G, x] + nx^{(n-1)(m+1)}CD[H, x]$. This completes the proof. Q

Theorem4. *If G be a graph having two components G_1 and G_2 with n and m vertices respectively, then*

$$CD[G, x] = \sum_{i=\delta(G_1)}^{\Delta(G_1)} Cd_i(G_1)x^{i+m} + \sum_{i=\delta(G_2)}^{\Delta(G_2)} Cd_i(G_2)x^{i+n}.$$

Proof. Let $v \in V(G_1)$ and $u \in V(G_2)$ be the vertices of G with degree d_1 and d_2 respectively. Since v is adjacent to $n - 1 - d_1$ vertices in G_1, then v is adjacent to $n-1-d_1+m$ vertices in G. Therefore, $deg(v) = n - 1 - d_1 + m$ in G. Similarly $deg(u) = n - 1 - d_1 + n$ in G. Since v and u are arbitrary vertices in G_1 and G_2 respectively, then degree of any vertex v in G_1 is $(deg(v)$ in $G_1)+m$ in G. Similarly degree of any vertex u in G_2 is $(deg(u)$ in $G_2)+n$ in G. Hence the result follows. Q

Corollary 5. *Let G be a graph having m components $G_1, G_2, \ldots, G_m$ where $|V(G_i)| = n_i$ for $i = 1, 2, \ldots, m$. Then*

$$CD[G, x] = \sum_{k=1}^{m} \sum_{i=\delta(\overline{G_k})}^{\Delta(\overline{G_k})} Cd_i(G_k)\, x^{\alpha}, \quad \text{where } \alpha = (i + \sum_{r=1, r\neq k}^{m} n_r).$$

Proof. The proof follows from theorem using mathematical induction on thenumber of components m of G.

The Mycielski graph, $\mu(G)$ of a graph G contains G itself as an isomorphic subgraph together with $n + 1$ additional vertices; a vertex u_i corresponding to each vertex v_i of G and another w. Each u_i is connected by an edge to w and for each edge v_iv_j of G, $\mu(G)$ includes two additonal edges v_iu_j andu_iv_j.

Theorem 6. *If G is a graph with n vertices, then we have* $CD[\mu(G), x] = x^2CD[G, x^2] + x^nCD[G, x] + x^n$.

Proof. Let $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, w$ be the vertices of $\mu(G)$(where $v_1, v_2, \ldots, v_n$ be the vertices of G, u_i corresponding to each v_i and w isthe vertex adjacent to each u_i, $i = 1, 2, \ldots, n$). Note that each v_i adjacentto $2(V(G) - N(v_i))$ vertices, u_i and w in $\mu(G)$. That is if $deg(v_i) = d$ in $\mu(G)$, then $deg(v_i) = 2(n-1-d) + 2$ in $\mu(G)$. Similarly each u_i adja- cent to $V(G) - N(v_i)$ vertices, $u_1, u_2, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n$ and v_i. That is $deg(u_i) = (n - 1 - d) + n - 1 + 1 = n-1-d+n$ in $\mu(G)$. Finally the vertex w adjacent to $v_1, v_2, \ldots, v_n$ in $\mu(G)$. That is $deg(w) = n$ in $\mu(G)$. Therefore, $CD[\mu(G), x] = x^2CD[G, x^2] + x^nCD[G, x] + x^n$. This completes the proof. Q

The shadowgraph $Sh(G)$ of a graph G is obtained by taking two copies of G, say G_1 and G_2 and joining each vertex of G_1 to the neighbors of the corresponding vertex of G_2.

Theorem 7. *If G is a graph with n vertices, the complement degree poly- nomial of the shadowgraph of G is given by* $CD[Sh(G), x] = 2xCD[G, x^2]$.

Proof. Let G_1 and G_2 be the two copies of G in $Sh(G)$ and let $v \in G_1$ and $deg(v) = d$ in G, then v adjacent to $V(G_1) - N(v)$, $V(G_2) - N(v)$ and v G_2 in $Sh(G)$. That is degree of v in $Sh(G)$ in n $1 + d + n$ $1 + d + 1 =$

$2(n\ 1\ d)+1$. It follows that $CD[Sh(G), x] = 2xCD[G, x^2]$. This completes the proof.

Duplication of a vertex v of a graph G is the graph denoted by G' obtainedby adding a vertex v' in G with $N(v) = N(v')$.

Theorem 8. *If $n \geq 1$, we have the following*

$$CD[K_n', x] = 2x + (n-1).$$

Proof. Let v $V(K_n)$ and v' $V(K_n')$ where v' be the duplicate vertex of v. Then v' adjacent to all vertices in K_n other than v. Thus v' adjacent to onlythe vertex v in K_n'. Similiarly v adjacent to only v' in K_n'. All other $n - 1$ vertices are isolated vertices in K_n'. Therefore, $CD[K_n', x] = 2x + (n\ \ 1)$. This completes the proof. Q

Theorem 9. *If V_1 and V_2 are the first and second set of vertices of $K_{m,n}$ with m and n order respectively and v' is the duplication of a vertex v of $K_{m,n}$, then*

$$CD[K'_{m,n}, x] = \begin{cases} (m+1)x^m + nx^{n-1}, & if v \in V_1 \\ mx^{m-1} + (n+1)x^n, & if v \in V_2 \end{cases}.$$

Proof. Let V_1 and V_2 be the first and second set of vertices of $K_{m,n}$ with m and n order respectively and v' be the duplication of a vertex v of $K_{m,n}$.Note that $K_{m,n}$ is an another complete bipartite graph $K_{m+1,n}$ if $v \in V_1$ or $K_{m,n+1}$ if $v \in V_2$. Therefore,

$$CD[K'_{m,n}, x] = \begin{cases} (m+1)x^m + nx^{n-1}, & if v \in V_1 \\ mx^{m-1} + (n+1)x^n, & if v \in V_2 \end{cases}.$$

This completes the proof. Q

A chaplet graph $C_p \bigcup C_q^t$ where $p, q.t \geq 3$ is obtained by taking one point union of t-copies of the cycle C_q and attaching the same to each vertex of the cycle C_p.

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Theorem 10.

$$CD[C_p \; K \; C_q^t, x] = px^{p(q-1)t+p-2t-3} + p(q-1)tx^{p(q-1)t+p-3}.$$

Proof. Let $u_1, u_2, \ldots, u_p$ be the vertices of the cycle C_p. For $j \in \{1, 2, \ldots, t\}$ and $k \in \{1, 2, \ldots, p\}$ let $u_k, u^j_{k_1}, u^j_{k_2}, \ldots, u^j_{k(q-1)}$ be the vertices of j^{th} copy of the cycle C_q attached to the vertex u_k of C_p. Note that degree of any vertices of C C^t other than $u, u, \ldots, u$ are 2 in C_p C^t. That is degree of these $p(q-1)t$ vertices are $p(q-1)t + p - 3$ in C_p C_q^t. But degree of $u_1, u_2, \ldots, u_p$ are $2t + 2$ in C_p C_q^t. Thus degree of $u_1, u_2, \ldots, u_p$ are $p(q-1)t - 3$ in C_p C_q^t. Therefore, $CD[C_p \; C_q^t, x] = px^{p(q-1)t+p-2t-3} + p(q-1)tx^{p(q-1)t+p-3}$. This completes the proof. Q

3. Results on Path graphs

Theorem 11. [3] *We have,*

$$CD[P_n, x] = \begin{cases} 2x^{n-2}, & n = 2 \\ (n-2)x^{n-3} + 2x^{n-2}, & n \geq 3. \end{cases}$$

The middle graph $M(G)$ of a graph G is the graph in which the vertex set is $V(G)$ $E(G)$ and two vertices are adjacent if and only if either they are adjacent edges of G or one is vertex of G and the other is an edge incident with it.

Theorem 12. *We have,*

$$CD[M(P_n), x] = \begin{cases} 2x + 1, & n = 2 \\ 2x^{2n-2} + (n-2)x^{2n-3} + 2x^{2n-4} + (n-3)x^{2n-5}, & n \geq 3. \end{cases}$$

Proof. If $n = 2$, then $M(P_2)$ is a path graph P_3. Thus $CD[M(P_2), x] = CD[P_3, x] = 2x+1$. When $n > 2$, since P_n has $n-1$ edges, $M(P_n)$ has $2n-1$ vertices say $v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_{n-1}$ where $v_1, v_2, \ldots, v_n$ be the vertices of P_n and $e_1, e_2, \ldots, e_{n-1}$ be the edges of P_n. Note that e_1 adjacent to e_2, v_1 and v_2. Similarly e_{n-1} adjacent to e_{n-2}, v_{n-1} and v_n. Thus $deg(e_1) = deg(e_{n-1}) = 2n - 1 - 3 = 2n - 4$ in $M(P_n)$. Since v_1 and v_n are adjacent to only e_1 and e_{n-1} respectively. Therefore, $deg(v_1) = deg(v_n) = 2n - 2$ in $M(P_n)$. The vertex v_i adjacent to e_{i-1} and e_i, then $deg(v_i) = 2n - 3, i = 2, 3, \ldots, n - 1$ in $M(P_n)$. The remaining vertices e_i, $i = 2, 3, \ldots, n - 2$ adjacent to $e-i-1$, e_{i+1}, v_i and v_{i+1}, then $deg(e_i) = 2n-5$, $i = 2, 3, \ldots, n-2$ in $M(P_n)$. Hence the result follows. Q

For a graph G the splitting graph $S(G)$ of graph G is obtained by addinga new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Theorem 13. *If $n \geq 2$, we have the following*

$$CD[S(P_n), x] = \begin{cases} 2x + 2x^2, & n = 2 \\ 2x^{2n-2} + nx^{2n-3} + (n-2)x^{2n-5}, & n \geq 3. \end{cases}$$

Proof. Since for $n = 2$, $S(P_2)$ is another path graph P_4, then $CD[S(P_2), x] = CD[P_4, x] = 2x + 2x^2$. When $n > 2$, let $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$ be the vertices of $S(P_n)$ where $v_1, v_2, \ldots, v_n$ be the vertices of P_n and u_i corresponding to v_i in $S(P_n)$, $i = 1, 2, \ldots, n$. Since v_1 and v_n are pendant vertices in P_n, then u_1 and u_n are pendant vertices in $S(P_n)$. Therefore, $deg(u_1) = deg(u_n) = 2n - 2$ in $S(P_n)$. Note that $deg(v_i) = 2, i = 2, 3, \ldots, n - 1$ in P_n, then $deg(u_i) = 2, i = 2, 3, \ldots, n - 1$ in $S(P_n)$. Thus $deg(v_i) = 2n - 3$, $i = 2, 3, \ldots, n - 1$ in $S(P_n)$. Also $deg(v_1) = deg(v_n) = 2$ in $S(P_n)$, then $deg(v_1) = deg(v_n) = 2n - 3$ in $S(P_n)$. Similarly the $deg(v_i) = 4$ in $S(P_n)$. Then $deg(v_i) = 2n - 5$, $i = 2, 3, \ldots, n - 1$ in $S(P_n)$. Therefore, $CD[S(P_n), x] = 2x^{2n-2} + nx^{2n-3} + (n\ 2)x^{2n-5}$. This completes the proof. Q

Theorem 14. *If G_n be a graph of order n, then $CD[S(G_n), x]$ do not have a constant term.*

Proof. Let $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$ be the vertices of $S(G_n)$ where $v_1, v_2, \ldots, v_n$ be the vertices of G_n and u_i corresponding to v_i in $S(G_n)$, $i = 1, 2, \ldots, n$. Note that u_i not adjacent to $v_i, i = 1, 2, \ldots, n$ in $S(G_n)$. Therefore, $\Delta(G) \leq 2n - 2$. This implies that there is no isolated vertex in $S(G_n)$. Thus $CD[S(G_n), x]$ do not have a constant term. This completes the proof. Q

The cosplitting graph $CS(G)$ is the graph obtained from G, by adding a new vertex w_i for each vertex v_i and joining w_i to all vertices which are notadjacent to v_i in G.

Theorem 15. *We have*

$$CD[CS(P_n), x] = \begin{cases} 2x + 2x^2, & n = 2 \\ nx^{n-1} + 2x^n + (n-2)x^{n+1}, & n \geq 3. \end{cases}$$

Proof. Since for $n = 2$, $CS(P_2)$ is another path graph P_4, then $CD[CS(P_2), x] = CD[P_4, x] = 2x + 2x^2$. When $n > 2$, let $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$ be the vertices of $CS(P_n)$ where $v_1, v_2, \ldots, v_n$ be the vertices of P_n and u_i corresponding to v_i in $CS(P_n)$, $i = 1, 2, \ldots, n$. Since $deg(v_i) = n$ in $CS(P_n)$,

$i = 1, 2, \ldots, n$. Therefore, $deg(v_i) = n - 1$ in $CS(P_n)$, $i = 1, 2, \ldots, n$. The vertex u_1 adjacent to each u_i, $i = 2, \ldots, n$ and v_2 in $CS(P_n)$. Thus $deg(u_1) = n - 1 + 1 = n$ in $CS(P_n)$. Similarly the degree of u_n is n in $CS(P_n)$. The remaining vertices u_i, $i = 2, 3, \ldots, n-1$ adjacent to each u_j, $j = 1, \ldots, i-1, i+1, \ldots, n$, v_{i-1} and v_{i+1}. Therefore, $deg(u_i) = n - 1 + 2 = n + 1$, $i = 2, 3, \ldots, n - 1$ in $CS(P_n)$. Hence for $n > 2$, $CD[CS(P_n), x] = nx^{n-1} + 2x^n + (n-2)x^{n+1}$. This completes the proof. Q

The derivative of a graph G is a graph $d(G)$ obtained from G by deletingall the pendant vertices of G.

Theorem 16. *If $n \geq 5$, we have the following $CD[d(P_n), x] = CD[P_{n-2}, x]$.*

Proof. Note that the derivative of a path graph P_n is again a path graph with n 2 vertices. Thus $CD[d(P_n), x] = CD[P_{n\ 2}, x]$. This completes the proof. Q

Theorem 17. *If G has no pendant vertices, then $CD[G, x] = CD[d(G), x]$.*

Proof. Let G be a graph and G has no pendant vertices then $G \cong d(G)$. Therefore, $CD[G, x] = CD[d(G), x]$. This completes the proof. Q

Theorem 18. *If n 3 and v' is a duplication of the pendant vertex of P_n, then*

$$CD[P^J_n, x] = x^{n-3} + (n-3)x^{n-2} + 3x^{n-1}.$$

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n and v' be the duplication of v_1, then v_1 is a pendant vertex. That is v_1, v_2 and v' are pendant vertices of P^J_n. Thus the degree of these 3 vertices is $n-1$ in P^J_n. Since v_1, v_3 and v' are adjacent to v_2, then $deg(v_2) = 3$ in P^J_n. Thus $deg(v_2) = n-3$ in P^J_n. The other $n-3$ inner vertices of P_n has degree 2 in P^J_n, then degree of that $n-3$ vertices are n 2 in P^J_n. Therefore, $CD[P^J_n, x] = x^{n-3} + (n\ 3)x^{n-2} + 3x^{n-1}$. This completes the proof.

Theorem 19. *If n 4 and v' is a duplication of the vertex of P_n which is not a pendant vertex but the neighbor of a pendant vertex, then*

$$CD[P^J_n, x] = x^{n-3} + (n-1)x^{n-2} + x^{n-1}.$$

Proof. Let v be the pendant vertex of P_n, $u \in N(v)$ and let v' be the duplication of u, then P^J_n form the tadpole graph $T_{3,n-2}$. Note that $CD[T_{m,n}, x] =$

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$x^{m+n-2}+(m+n\quad 2)x^{m+n-3}+x^{m+n-4}$. Thus $CD[P_n^J, x] = x^{3+n-2-2} + (3+n\ 2\ 2)x^{3+n-2-3} + x^{3+n-2-4} = x^{n-1} + (n\quad 1)x^{n-2} + x^{n-3}$. This completesthe proof.

Theorem 20. *If $n \geq 5$ and v^J is a duplication of the inner vertex of P_n which is not a neighbor of pendant vertex, then*

$CD[P_n^J, x] = 2x^{n-3} + (n-3)x^{n-2} + 2x^{n-1}$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n and v^J be the duplication of the inner vertex v_i, $i = 3, 4, \ldots, n-2$, then $deg(v_{i-1}) = deg(v_{i+1}) = 3$ in P_n^J. Thus $deg(v_{i-1}) = deg(v_{i+1}) = n - 3$ in P_n^J. Note that $deg(v_1) = deg(v_n) =$

$n-1$ in P_n^J and the degree of other $n-3$ vertices is $n-2$ in P_n^J including v^J

other than $v_1, v_n, v_{i-1}, v_{i+1}$. Therefore, $CD[P_n^J, x] = 2x^{n-3} + (n3)x^{n-2} + 2x^{n-1}$. This completes the proof.

A graph $L(G)$ is said to be a line graph of an undirected simple graph$G$ if the vertex set of $L(G)$ is in one-one correspondence with the edge setof G and two vertices of $L(G)$ are joined by an edge if and only if the correspondence edge of G are adjacent in G.

Theorem 21. *If $n \geq 3$, we have the following*

$$CD[L(P_n), x] = \begin{cases} 2x^{n-3}, & n = 3 \\ (n-3)x^{n-4} + 2x^{n-3}, & n \geq 4. \end{cases}$$

Proof. Since the line graph of P_n is another path graph P_{n-1}. Therefore,

$CD[L(P_n), x] = CD[P_{n-1}, x]$. This completes the proof. Q

Total graph denoted by $T(G)$ of a graph G is a graph in which the setof vertices and edge set of G and any two vertices in $T(G)$ are said to beadjacent if and only if their corresponding elements are either adjacent or incident in G.

Theorem 22. *We have,*

$$CD[T(P_n), x] = \begin{cases} 3, & n = 2 \\ 2x^{2n-4} + 2x^{2n-5} + (2n-5)x^{2n-6}, & n \geq 3. \end{cases}$$

Proof. Since $T(P_2)$ is C_3, then $CD[T(P_2), x] = CD[C_3, x] = 3$. When $n \geq 3$, let $v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_{n-1}$ be the vertrices of $T(P_n)$ where $v_1, v_2, \ldots, v_n$ be the vertices of P_n and $e_1, e_2, \ldots, e_{n-1}$ be the edges of P_n. Since v_1 adjacent to v_2 and e_1 in $T(P_n)$, $deg(v_n) = 2n - 4$ in $T(P_n)$. Sim- ilarly $deg(v_n) = 2n - 4$ in $T(P_n)$. Note that e_1 adjacent to v_1, v_2 and e_2, then $deg(e_1) = 2n - 5$ in $T(P_n)$. Similarly $deg(e_{n-1}) = 2n - 5$ in $T(P_n)$. The degree of remaining vertices $v_2, \ldots, v_{n-1}, e_2, \ldots, e_{n-2}$ are 4 in $T(P_n)$,

then degree of these $2n\quad 5$ vertices are $2n - 6$ in $T(P_n)$. Therefore, for $n \geq 3$, $CD[T(P_n), x] = 2x^{2n-4} + 2x^{2n-5} + (2n\quad 5)x^{2n-6}$. This completes the proof.

The *central graph* denoted by $C(G)$ of a graph G is the graph obtainedby subdividing each edge of G exactly once and joining all the non-adjacent vertices of G.

Q

Theorem 23. *If $n \geq 2$, we have the following* $CD[C(P_n), x] = (n-1)x^{2n-3} + nx^{n-1}$.

Proof. Let $v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_{n-1}$ be the vertices of $C(P_n)$ where v_1, v_2

$, \ldots, v_n$ be the vertices of P_n and $e_1, e_2, \ldots, e_{n-1}$ be the new vertices for sub-dividing each edge. Since the degree of each new vertices e_i,$i = 1, 2, \ldots, n-1$ is 2 in $C(P_n)$, $deg(e_i) = 2n - 4$ in $C(P_n)$. Note that v_1 adjacent to $v_3, v_4, \ldots, v_n$ and e_1 in $C(P_n)$, that is $deg(v_1) = n - 1$ in $C(P_n)$. Then $deg(v_1) = 2n - 1 - 1 - (n-1) = n - 1$ in $C(P_n)$. Similiarly $deg(v_n) = n - 1$in $C(P_n)$. But the remaining vertices v_i,$i = 2, 3, \ldots, n-1$ adjacent to $v_1, \ldots, v_{i-2}, v_{i+2}, \ldots, v_n, e_{i-1}$ and e_i, that is $deg(v_i) = n - 3 + 2 = n - 1$ in $C(P_n)$. Therefore, $deg(v_i) = n\quad 1$, $i = 2, 3, \ldots, n\quad 1$ in $C(P_n)$. Hence

$CD[C(P_n), x] = (n\quad 1)x^{2n-3} + 2x^{n-1} + (n\quad 2)x^{n-1} = (n\quad 1)x^{2n-3} + nx^{n-1}$. This completes the proof.

4. Results on regular graphs

Theorem 24. *Let G be the r-regular graph of order n, then*

$CD[S(G), x] = nx^{(2n-1-r)} + nx^{(2n-1-2r)}$.

Proof. Let $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n$ be teh vertices of splitting graph $S(G)$of regular graph G where $v_1, v_2, \ldots, v_n$ be the vertices of G and $u_1, u_2, \ldots, u_n$ be the corresponding vertices $v_1, v_2, \ldots, v_n$. Note that u_i, $i = 1, 2, \ldots, n$ adjacent to $n - r$ vertices in $\{v_1, v_2, \ldots, v_n\}$ and $n - 1$ vertices $u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n$ in $S(G)$. That is, $deg(u_i) = n-1+n-r = 2n-1-r$ in $S(G)$. But degree of v_i, $i = 1, 2, \ldots, n$ in $S(G)$ twice the degree in G.Therefore, degree of

each v_i is $2n\ 1\quad 2r$ $S(G)$. Hence $CD[S(G), x] = nx^{(2n-1-r)} + nx^{(2n-1-2r)}$. This completes the proof.

Theorem 25. *Let G be an r-regular graph of order n, then*

$CD[CS(G), x] = nx^{n-1}(1 + x^r)$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the regular graph G and $w_1, w_2,$

$\ldots, w_n$ be the new vertices of $CS(G)$. Let $v_i \in V(G)$. Then $deg(v_i) = n$ in $CS(G)$. This implies that $deg(v) = n - 1$ in $CS(G)$. Since each w_i adjacent to $n-r$ vertices in $CS(G)$, we have w_i adjacent to r vertices in $V(G)$ and $n-1$ vertices $w_1, w_2, \ldots, w_{i-1}, w_{i+1}, \ldots, w_n$ in $CS(G)$. That is $deg(w_i) = n-1+r$ in $CS(G)$. Therfore, $CD[CS(G), x] = nx^{n-1} + nx^{n-1+r} = nx^{n-1}(1 + x^r)$.. This completes the proof.

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Theorem 26. *If G is a r-regular graph of order n, then*

$$CD[G^J, x] = (n + 1 - r)x^{n-r} + rx^{n-r-1}.$$

Proof. Let G be the r- regular graph of order n and let v^J be the duplicationof a vertex v in G. Note that v^J adjacent to r vertices in G^J. But then v^J adjacent to $n - r$ vertices in G^J. Similarly, the vertices in $V - N(v)$ in G are adjacent to $n \quad r$ vertices in G^J. The vertices in $N(v)$ in G are adjacent to r vertices in G and v^J. This implies that the degree of vertex in $N(v)$ in G is $r + 1$ in G^J. Then degree of that r vertices is $n - r - 1$ in G^J. Therefore, $CD[G^J, x] = (n + 1 - r)x^{n-r} + rx^{n-r-1}$. This completes the proof.

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