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CORRECTION OF THE TRAJECTORY FOR UAV ACCORDING TO THE CRITERION OF GENERALIZED WORK WITH A PREDICTIVE MODEL

BY

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Abstract:

Consider an optimal control synthesis algorithm based on the dynamic programming method, using a special form of the optimized functional, which is called the generalized work criterion. When using the generalized work criterion, the partial differential equation that is used to select the optimal control becomes linear, which greatly simplifies the solution of the synthesis problem. The article refers to the synthesis problem optimal control laws UAV based on the dynamic programming method.

Keywords: *Air UAV; rudder gas; control laws; optimal control laws*

1. Introduction

To synthesize the optimal control law for UAV, we need to know the complete system of equations describing the air floor's motion as a control object in space. By ignoring the problems of durability, deformation, and oscillation of the UAV structure, only interested in the movement of the mass centre of the air floor and its rotation around the centre of mass, we can the problem limit in the motion of a solid body has 6 degrees of freedom (including three translational movements and three rotations). In addition, when ignoring the influence of moments generated by the dynamic systems and considering only the influence of aerodynamic forces and moments in combination with the above assumptions, we can show that most fully show the motion of missilion in the vertical plane by the following seven differential equations[1,4]:

$$\begin{cases} m\dfrac{dV}{dt} = P\cos\alpha - X - G\sin\theta \\ mV\dfrac{d\theta}{dt} = \mathrm{P}\sin\alpha + \mathrm{Y} - G\cos\theta \\ J_z\dfrac{d\omega_z}{dt} = \sum M_z \\ \dfrac{dy}{dt} = V\sin\theta \\ \dfrac{dx}{dt} = V\cos\theta \\ \dfrac{d\vartheta}{dt} = \omega_z \\ \theta = \vartheta - \alpha \end{cases} \qquad (1)$$

Suppose we consider the variation in mass m of the air floor to be insignificant during the operation of the engine operation, the UAV has sufficient static stability (angle of attack is always very small and angle) The above equation is converted to:

$$\begin{cases} m\dfrac{dV}{dt} = P - X - G\sin\theta \\ mV\dfrac{d\theta}{dt} = \mathrm{P}\alpha + \mathrm{Y} - G\cos\theta \\ \dfrac{dy}{dt} = V\sin\theta \\ \dfrac{dx}{dt} = V\cos\theta \end{cases} \qquad (2)$$

To complete the equation for the UAV input state as required, we need to add a control equation. According to [3], the lead methods aircraft using the dynamic throttle blade is implemented according to the proportional guide method. The method's essence is to ensure that the overload of the UAV is proportional to the angular speed of the line of sight. Therefore, the equation for the input status of an missilon is shown as follows [1]:

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$$\begin{cases}\dfrac{dV}{dt}=\dfrac{P}{m}-\dfrac{X}{m}-g\sin\theta\\ \dfrac{d\theta}{dt}=\dfrac{P\alpha}{mV}+\dfrac{g}{V}(\dfrac{Y}{mg}-\cos\theta)\\ \dfrac{dy}{dt}=V\sin\theta\\ \dfrac{dx}{dt}=V\cos\theta\\ \dfrac{dn_y}{dt}=u\end{cases}$$

Instead, $X=C_x\dfrac{\rho V^2}{2}S$ We have:

$$\begin{cases}\dfrac{dV}{dt}=\dfrac{P}{m}-C_x\dfrac{\rho V^2}{2}\dfrac{S}{m}-g\sin\theta\\ \dfrac{d\theta}{dt}=\dfrac{P\alpha}{mV}+\dfrac{g}{V}n_y-\dfrac{g}{V}\cos\theta\\ \dfrac{dy}{dt}=V\sin\theta\\ \dfrac{dx}{dt}=V\cos\theta\\ \dfrac{dn_y}{dt}=u\end{cases}\tag{3}$$

Inside: $\rho=1.225(1-\dfrac{H}{44300})^{4.256}$ - air density at altitude H;

Thus, the problem is explicitly stated as follows:

Determine the optimal control law u (t) to take UAV with the mathematical model (3) from a first state point $\underline{x}(0)=\underline{x}_0$ arbitrarily given to the end state point $\underline{x}_T$ given in the fastest period satisfying the function target:

$$J=\frac{1}{2}(\rho_{11}(x-x_{giv})^2+\rho_{22}(y-y_{giv})^2)\tag{4}$$

With ρ_{11},ρ_{22},k given coefficients determined from the principle of equal contributions of maximum deviations and x_{giv}, y_{giv} - gien value.

With the top conditions:

$$V(0)=V_0,\theta(0)=\theta_0,y(0)=y_0,x(0)=x_0,$$
$$n_y(0)=n_{y0},|n_y|\le n_{ygioihan}$$

2. Principle of the criterion of generalized work with a predictive model

Let the motion of the controlled system be determined by the following differential vector equation:

$$\frac{dx}{dt}=f(t,x)+F(t,x)u\tag{5}$$

With: $x=\begin{bmatrix}x_1\\x_2\\.\\.\\.\\x_n\end{bmatrix}$; $f(t,x)=\begin{bmatrix}f_1(t,x)\\f_2(t,x)\\.\\.\\.\\f_1(t,x)\end{bmatrix}$;;

$$F(t,x)=\begin{bmatrix}\varphi_{11}(t,x)...\varphi_{1m}(t,x)\\ \varphi_{21}(t,x)...\varphi_{2m}(t,x)\\ .\quad .\quad .\\ .\quad .\quad .\\ .\quad .\quad .\\ \varphi_{n1}(t,x)...\varphi_{nm}(t,x)\end{bmatrix}\quad u=\begin{bmatrix}u_1\\u_2\\.\\.\\.\\u_n\end{bmatrix}$$

The initial conditions are given:

$$x^T(t_0)=x_0^T=(x_{10},x_{20},...,x_{n0})$$

Let the right end of the trajectory be free, and the moment of the end of the transient process ϑ be given. There are no restrictions on the control vector. It is required to find the optimal control under which the criterion of generalized operation takes the minimum value

$$J=R(x(\vartheta))+\int_{t_0}^{\vartheta}Q(t,x)dt+\frac{1}{2}\int_{t_0}^{\vartheta}\sum_{j=1}^{m}\left(\frac{u_j}{k_j}\right)^2dt+\frac{1}{2}\int_{t_0}^{\vartheta}\sum_{j=1}^{m}\left(\frac{\tilde{u}_j}{k_j}\right)^2dt\tag{6}$$

With $\tilde{u}_j=-k_j^2\left(\sum_{k=11}^{n}\varphi_{kj}\dfrac{\partial V}{\partial x_k}\right)$ (7)

V-solution of a linear partial differential equation

$$\frac{\partial V}{\partial t}=\left(\frac{\partial V}{\partial t}\right)^T f(t,x)=-Q$$

(8)

under boundary conditions $V(\vartheta)=R(x(\vartheta))$. (9)

The criterion of generalized operation differs from the criterion usually used in the synthesis of control systems, only by the presence of the last term, which takes into account the so-called generalized operation of control signals. Let us show that the control (7) is optimal for the problem posed, and the equation for determining the V-function has the form (8).

Let us compose the Bellman equation for system (5) with the criterion of generalized work:

$$-\frac{\partial V}{\partial t}=\min_u\left[Q(t,x)+\frac{1}{2}\sum_{j=1}^{m}\left(\frac{u_j^2+\tilde{u}_j^2}{k_j^2}\right)+\left(\frac{\partial V}{\partial x}\right)^T f(t,x)+\left(\frac{\partial V}{\partial x}\right)^T F(t,x)u\right]$$

(10)

Let's find the derivative of the right side with and equate it to zero, then

$$\sum_{j=1}^{m}\left[\frac{u_j}{k_j^2}\sum_{k=1}^{n}\varphi_{kj}\frac{\partial V}{\partial x_k}\right]=0$$

Where

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$$u_j = -k_j^2 \sum_{k=1}^{n} \phi_{kj} \frac{\partial V}{\partial x_k}. \qquad (11)$$

Substitute (11) into (10), then equation (10) will have the form

$$-\frac{\partial V}{\partial t} = Q(t,x) + \left(\frac{\partial V}{\partial x}\right)^T f(t,x) + \frac{1}{2}\sum_{j=1}^{m}\left[k_j^2\left(\sum_{k=1}^{n}\varphi_{kj}\frac{\partial V}{\partial x_k}\right)^2\right] + \frac{1}{2}\sum_{j=1}^{m}\frac{\tilde{u}_j^2}{k_j^2} - \sum_{j=1}^{m}\left[\left(\sum_{k=1}^{n}\varphi_{kj}\frac{\partial V}{\partial x_k}\right)k_j^2\left(\sum_{k=1}^{n}\varphi_{kj}\frac{\partial V}{\partial x_k}\right)\right]$$

If we take it in the form (10), then the equation for determining the V - function will look like (8). Thus, the optimal control for this problem has the form (7). To determine the optimal control, it is necessary to know the partial derivatives

$$\frac{\partial V}{\partial x_k} \quad (k = 1,2,...,n)$$

An algorithm with $\frac{\partial V}{\partial x_k}$ a predictive model is used to calculate partial derivatives and select the optimal control in the process of moving along the trajectory.

When moving with equation (5) has the form $\frac{dx}{dt} = f(t,x)$, then the left side of equation (8)

$$\frac{\partial V}{\partial t} + \left(\frac{\partial V}{\partial t}\right)^T f(t,x) = -Q$$

turns into a total derivative with respect to time: $\frac{dV}{dt} = -Q$

It follows from this that $V(x_M(\vartheta)) - V(x_M(t_0)) = -\int_{t_0}^{\vartheta} Q(t,x_M)dt$

Here the index "M" denotes the movement corresponding to and satisfying the equation

$$\frac{dx_M}{dt} = f(t,x_M) \qquad (12)$$

The motion corresponding to equation (12) is called a predictive model.

When controlling real systems, equation (12) must be integrated in accelerated time $\tau = t/k$, where $k = const >> 1$.

In this case, the predictive model equation will look like

$$\frac{dx_M}{dt} = kf(\tau k, x_M)$$

When managing real systems, it should be tens, hundreds, and sometimes thousands of units.

Below we will assume that. For an arbitrary initial moment $t(t < \vartheta)$, taking into account (9), we can write:

$$V(x_M(t)) = R(x_M(\vartheta)) + \int_t^{\vartheta} Q(t,x_M)dt \qquad (13)$$

Thus, to determine the function $V(t)$ at the moment t, it is necessary to solve system (12) with initial conditions $x_M(t) = x(t)$ on the time interval $[t,\vartheta]$, determine $x_{M\vartheta} = x_M(\vartheta)$, and then integrate in the opposite direction of time from ϑ to t equation (13) with boundary conditions (9) and the equation predictive model (12) with boundary conditions $x_M(\vartheta) = x_{M\vartheta}$.

To determine partial derivatives $\frac{\partial V}{\partial x_i}(t)$, numerical differentiation methods can be used, but they require in this case n additional integrations of equation (13) in the backward direction of time.

An optimal control algorithm with a predictive model is assumed, which does not explicitly contain the numerical differentiation of the function $V(t)$.

We differentiate equation (8) with respect to x and introduce the notation

$$\Psi = \frac{\partial V}{\partial x} \qquad (14)$$

with $\Psi^T = (\Psi_1, \Psi_2, ..., \Psi_n)$ then

$$\frac{\partial \Psi}{\partial t} + \frac{\partial \Psi}{\partial x} f + \left(\frac{\partial f}{\partial x}\right)^T \Psi = -\frac{\partial Q}{\partial x} \qquad (15)$$

On the integral curves of the model equation (12), the partial differential equation (15) turns into an ordinary differential equation

$$\frac{\partial \Psi}{\partial t} = -\left(\frac{\partial f}{\partial x}\right)_M^T \Psi - \frac{\partial Q}{\partial x} \qquad (16)$$

where the index "M" means that the corresponding values are taken on the movement of the predictive model.

Since $V(\vartheta) = R(x_M(\vartheta))$ then from (14) the final conditions are obtained:

$$\Psi(\vartheta) = \left.\frac{\partial R}{\partial x}\right|_{x_M(\vartheta)} \qquad (17)$$

Thus, the optimal control synthesis algorithm according to the criterion of generalized work with a predictive model is as follows:

Step 1. To select the optimal control at the moment t_0 of time, the system of equations of the predictive model (12) is solved with initial conditions $x_M(t_0) = x_0$ from $t = t_0$ to $t = \vartheta$, and the vector $x_{M\vartheta} = x_M(\vartheta)$ is determined.

Step 2. The boundary conditions (16) are determined for the vector Ψ

Step 3. The system of equations (15) with the boundary conditions (16) and the system (12) with the boundary conditions $x_{M\vartheta} = x_M(\vartheta)$ are solved jointly in the reverse direction of time from $t = \vartheta$ to $t = t_0$

Step 4. The optimal control for the moment of time t_0 is determined by the formula (7)

$$\tilde{u}_j(t_0) = -k_j^2\left(\sum_{k=1}^{n}\varphi_{kj}(t_0,x_0)\Psi_k(t_0)\right)$$

Step 5. With this control, the system (5) moves through time Δt and moves to a new position $x(t_0 + \Delta t)$.

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Step 6. To select the optimal control at the moment of time $t_0 + \Delta t$, the equations of the predictive model are again solved with initial conditions $x_M(t_0 + \Delta t) = x(t_0 + \Delta t)$ from $t = t_0 + \Delta t$ to $t = \vartheta$, etc.

3. Conclusion

Thus, found the dependence of the dynamic priorities of each page on its relative importance, that is, on the number of new dangerous changes in the stress state, on the delay time in the presentation of the page on the screen, as well as on the expected degree of reliability of a structural element, or an a priori estimate of its safety margin.

The resulting dependence can be used to form examples of training a neural network that controls displaying pages during tests. The same network can be trained to recognize the stress plots of elements and diagnose failures during strength tests.

4. Synthesis of UAV control laws according to the criterion of generalized with a predictive model

We see that the input state equation (3) of the UAV can be rewritten as a vector equation:

$$\frac{dx}{dt} = f(\mathrm{t,x}) + \mathrm{F(t,x)\,u} \quad \text{with}$$

$$x = \begin{bmatrix} v \\ \Theta \\ y \\ x \\ n_y \end{bmatrix}; \quad f(t,x) = \begin{bmatrix} g(n_x - \sin\Theta) \\ \frac{g}{v}(n_x - \cos\Theta) \\ v\sin\Theta \\ v\cos\Theta \\ 0 \end{bmatrix}; \quad F(t,x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

Instead of the minimum of the criterion (4), we will look for the minimum of the generalized work functional:

$$J_1 = R(x(\vartheta)) + \int_{t_0}^{\vartheta} Q(t,x)dt \int_{t_0}^{\vartheta} \frac{u^2 + \tilde{u}^2}{k^2} dt \qquad (18)$$

With

$R(x(\vartheta)) = \frac{1}{2}(\rho_{11}(x - x_{giv})^2 + \rho_{22}(y - y_{giv})^2)$; $Q(t,x) = 0$,

System (15) in this case has the form:

$$\frac{d\Psi_v}{dt} = \left(\frac{gC_x\rho vS}{mg}\right)\Psi_v + \left(\frac{g\cos\Theta}{v^2}\right)\Psi_\Theta - \sin\Theta\Psi_y - \cos\Theta\Psi_x$$

$$\frac{d\Psi_\Theta}{dt} = g\cos\Theta\Psi_v - \frac{g\sin\Theta}{v}\Psi_\Theta - v\cos\Theta\Psi_y + v\sin\Theta\Psi_x$$

$$\frac{d\Psi_x}{dt} = 0$$

$$\frac{d\Psi_y}{dt} = 0$$

$$\frac{d\Psi n_y}{dt} = -\frac{g}{v}\Psi_\Theta$$

(19)

Boundary conditions (16) in this case are:

$$\Psi_v(\vartheta) = 0$$
$$\Psi_\Theta(\vartheta) = 0$$
$$\Psi_y = \rho_{22}(y - y_{giv})$$
$$\Psi_x = \rho_{11}(x - x_{giv})$$
$$\Psi n_y = 0$$

The predictive model $\frac{dx_M}{dt} = \kappa \mathrm{f}(\tau\kappa, x_M)$ will be written as follows:

$$\left.\begin{aligned} \frac{dv}{dt} &= \kappa g(n_x - \sin\Theta), \\ \frac{d\Theta}{dt} &= \kappa\frac{g}{v}(n_y - \cos\Theta), \\ \frac{dy}{dt} &= \kappa v\sin\Theta, \\ \frac{dx}{dt} &= \kappa v\cos\Theta, \\ \frac{dn_y}{dt} &= 0 \end{aligned}\right\} \qquad (20)$$

We will assume $\kappa = 1$, Optimal control $\tilde{u}(t,x)$ is determined by the formula:

$\tilde{u}(t,x) = -k^2\Psi n_y(t)$

In the passive section, the flight of a UAV projectile occurs without reactive force, in real conditions, due to the impact on the projectile of random factors of the spread of parameters, its movement will be perturbed. To improve the accuracy of the aircraft, motion control with trajectory correction is necessary.

5. Simulation and evaluation of the results

- Building the reachability area under the condition of using a predictive model from the highest point of the trajectory.

max V = 657.954 m/s

max T = 95.603 s

max X = 62417.211 m

max Y = 41412.857 m

max θ = 0.001^0

To simulate and evaluate the results for “ATACMS mod2”[8]. *With at max flight.*

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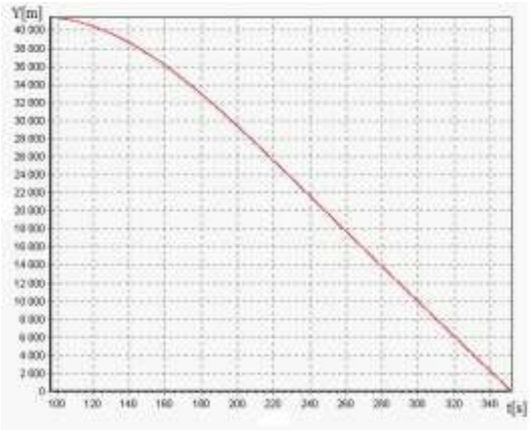
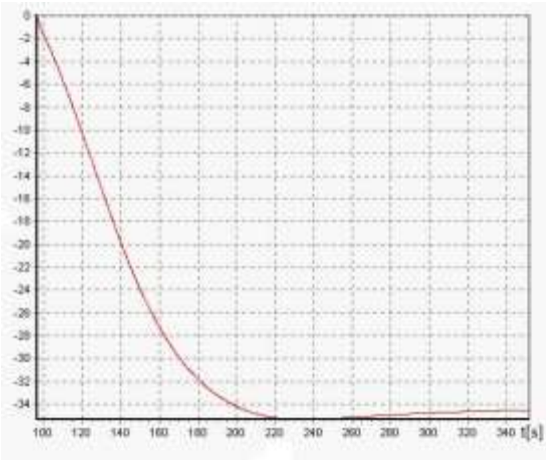
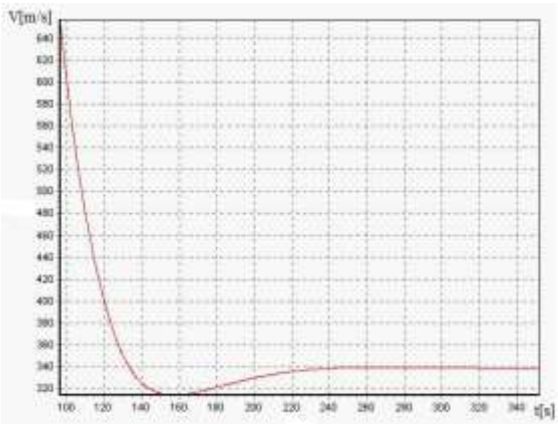
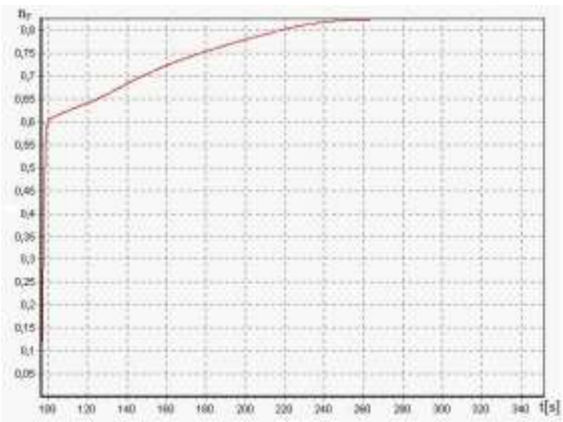
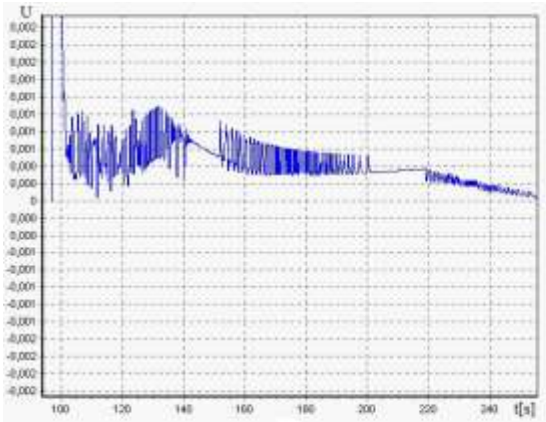


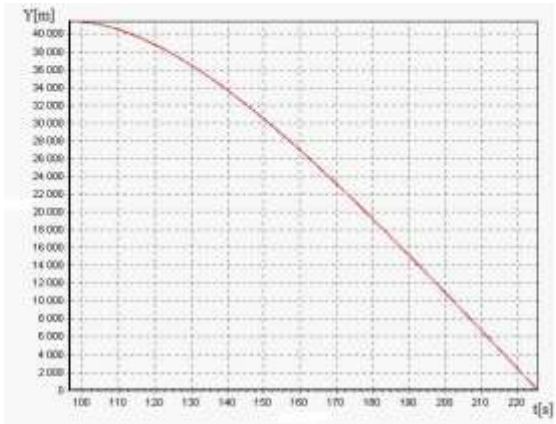


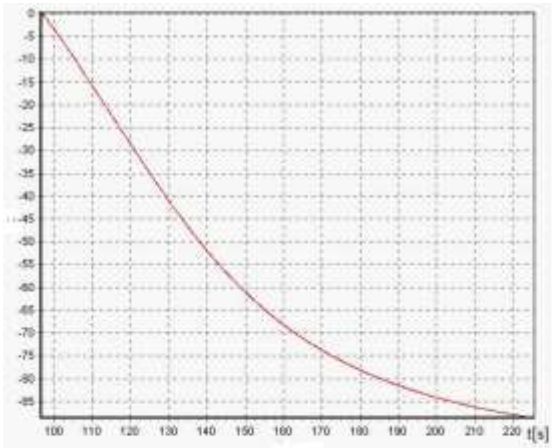


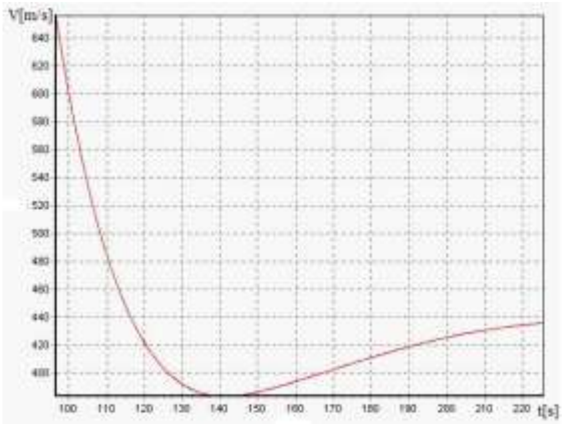




With at min flight.







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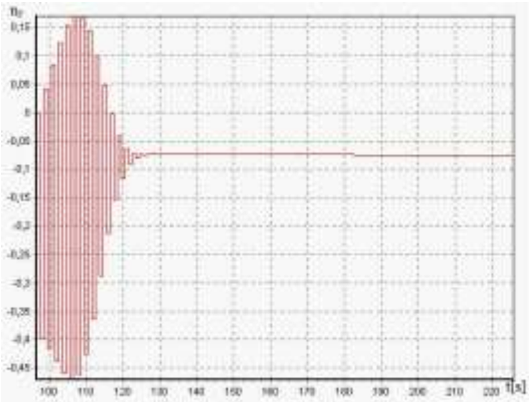
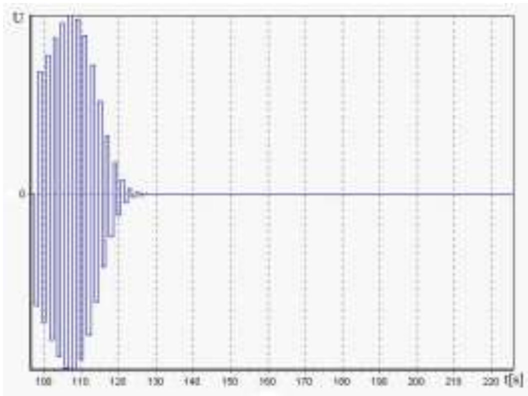
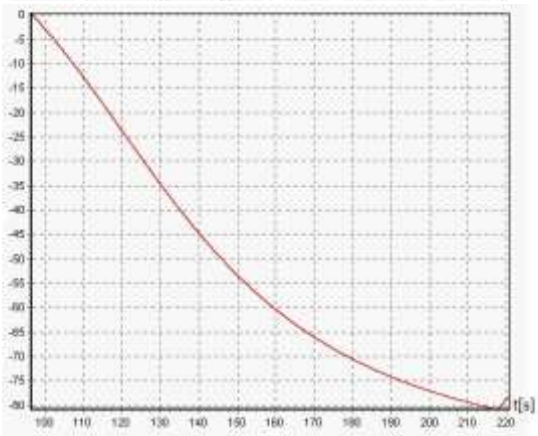
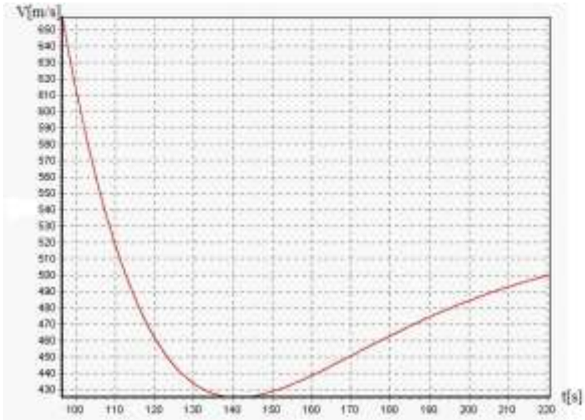


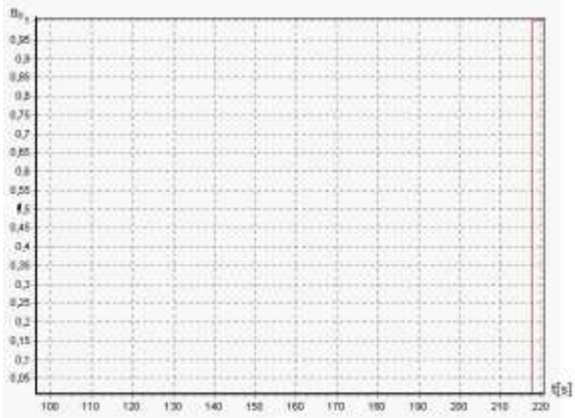


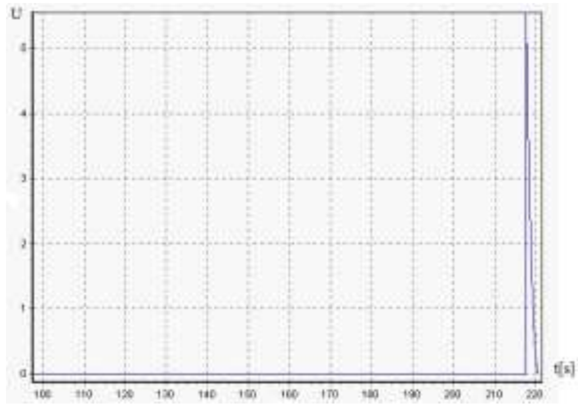
- Building a reachable area using a predictive model from a height of 1500 meters

With at max flight.









With at min flight.

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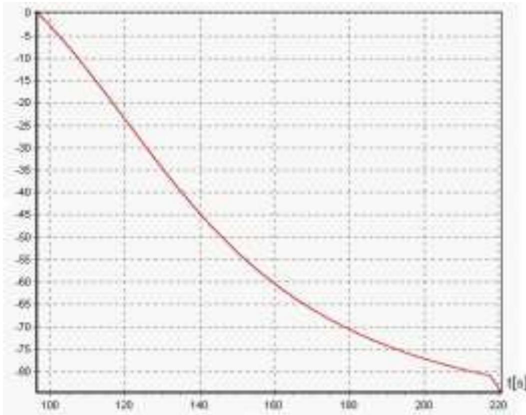

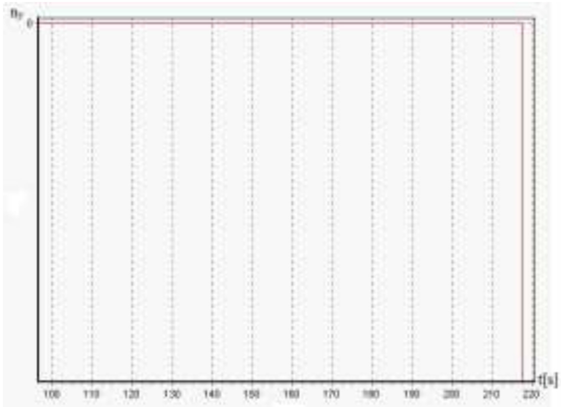

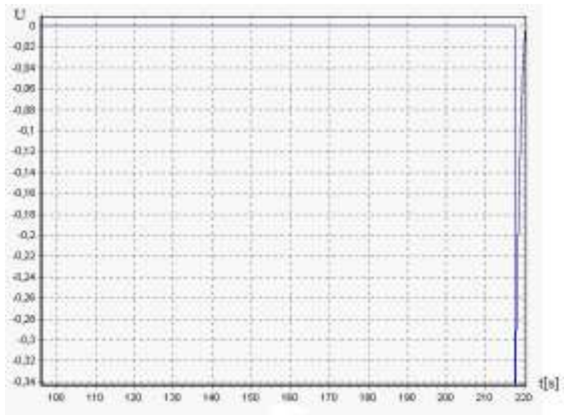

Discussion:

When considering the methods for constructing the reachability regions, the method for constructing the reachability regions was analyzed according to the criterion of generalized work with a predictive model. On the basis of this theory, a program was written that builds the trajectory of an aircraft, taking into account the variable control of the normal overload n_y. Two reachability regions are constructed:

- When using control from the highest point in the descending branch of the trajectory.

- When using control from a height of 1500 m.

In the first case, the reachability areas in the XY plane is 52000 m. In the second case, the reachability areas in the XY plane is 89.64 m

Thus, in this case, the optimal control signal of the air floor name according to the criterion of generalized work with a predictive model is relatively consistent with reality.

6. Conclusion

By calculating and evaluating the results, we see that the results achieved are consistent with the control process's physical nature. It is possible to use the criterion of generalized work with a predictive model to synthesize the optimal control law for air train name using the dynamic throttle as the basis for researching, designing and manufacturing air floor names with different purposes in practice.

References

1. R. Ashwini, "Analysis of two-stage proportional navigation with heading constraints" *Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 1, Pp. 156-164, 2015.
2. S.B. Majumder, "Practical aspects of optimal midcourse guidance for air-to-air engagement," *IFACPapersOnLine*, Vol. 49, No. 1, Pp. 591-596, 2016.
3. J. Byoung-Ju, B.G. Park, M.J. Tahk., "Optimal midcourse guidance law with flight path angle and lead angle constraints to reach circular target area," *IFAC Proceedings*, Vol. 46, No. 19, Pp. 336-340, 2013.
4. Z. Youan, G. Ma, A. Liu, "Guidance law with impact time and impact angle constraints," *Chinese Journal of Aeronautics*, Vol. 26, No. 4, Pp. 960-966, 2013
5. C. Xiaotian, J. Wang, "Optimal control-based guidance law to control both impact time and impact angle," *Aerospace Science and Technology*, Vol. 84, Pp. 454-463, 2019.
6. P. Bong-gyun, "Optimal impact angle constrained guidance with the seekers lock-on condition," *Journal of the Korean Society for Industrial and Applied Mathematics*, Vol. 19, No. 3, Pp. 289-303, 2015.
7. Y. Wenbin, "Optimal terminal guidance for exoatmospheric interception," *Chinese Journal of Aeronautics*, Vol. 29, No. 4, Pp. 1052-1064, 2016.

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